Full-wave modeling of multilayer superconducting microstrip lines sensitivities

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Abstract

This paper studies the sensitivities of superconductor microstrip line on multilayer substrate and superstrate using a new theoretical model. This model carefully considers the effects of superconductivity, the strip thickness and losses on circuit performances. The multilayer superconducting microstrip line has been considered as one directional bianistropic medium. Therefore, new integral equation for the electrical field is formulated, in the spectral domain, using the exact dyadic Green's function of a bianisotropic planar media. In order to calculate the geometrical microstrip sensitivities, we have used the finite difference method to obtain the total geometrical derivative of the electrical field integral equation. We have used the two-dimensional Galerkin's technique to solve the set of resulting integral equations. The computations of multilayer superconductor microstrip line are compared to those provided by the quasi-static and full wave approaches. However, our results of the geometrical sensitivities of the microstrip characteristics are compared only with quasi-static results, because there is no approaches in the literature developing such calculations.

Key words: Modeling, Superconductivity, Sensitivity, Galerkin's technique, Green's function, Multilayered microstrip.

1. Introduction

Considerable research has been devoted to the full-wave analysis of planar microstrip structures as can be seen from the numerous publications on this topic [1]. In order to develop some special microwave devices, the designers of new multilayer circuit boards use several techniques for improve the adhesion between the substrates and ground plane with temperature variation. One of these techniques, that based on the perforations in the ground plane [2]. A second technique uses ground planes fabricated by laying one layer of metallic wires over the substrate followed by an other layer of wires perpendicular to the first layer [2]. Hence, such ground plane, substrate and superstrate can be treated as three stacking layers of anisotropic media. An other approach uses the high-temperature superconductors characterized by their low surface resistance and frequency-independent penetration dept.

Such a rigorous analysis is very often based on an integral equation formulation, typically solved with the method of moments [1-9]. Some of these analyses incorporate the effect of superconducting into the electrical field integral equation formulation basing on the electrical surface current density on the electrical field [3].

In this paper, we have developed an original theoretical model to analyze the sensitivities of the propagation characteristics of superconducting microstrip line on lossy multilayer dielectric substrate. In this model, based on the exact dyadic Green's function of bianisotropic media [5-7], we formulate a set of integral equations in the spectral domain. We have applied a two-dimensional Galerkin's technique, to solve this last for the effective propagation constant and its derivative with respect to the geometrical parameter. In addition, we have extracted the effective permittivity and the characteristic impedance of the microstrip line and their geometrical derivative.

2. Theory

Consider the multilayer superconducting microstrip line shown in figure1. The substrate and superstrate consist of an arbitrary number of layers, stacked in the z-direction. The layers extend to infinity in x- and y-directions. The thickness of the metallization of the superconductor is considered. Then, the surface current is assumed to flow only in x-, and y-directions in the strip.

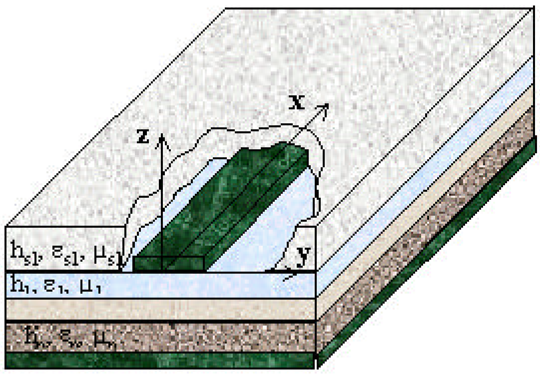

Fig. 1. Geometry of multilayer superconducting microstrip line.

2.1. Microstrip modeling

Using the Maxwell's formulation and applying the three-dimensional dyadic Green's function of a planar bianisotropic media, to the multilayer microstrip, yield the electrical field element integral equation given by [1-14]:

$$d\vec{E}(x,y,z) = \int_V \overline{\overline{G}}(\vec{r}/\vec{r}_0)\cdot\vec{J}(\vec{r}_0)dV_0 \quad (1)$$

where $\vec{J}(\vec{r}_0)$ is the three-dimensional current density, and $\overline{\overline{G}}(\vec{r}/\vec{r}_0)$ is the three-dimensional dyadic Green's function expressed as [5]:

$$\overline{\overline{G}}(\vec{r}/\vec{r}_0) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int \overline{g}(k_x, k_y, z/z_0) e^{jk_x(x-x_0)} e^{jk_y(y-y_0)} dk_x dk_y \tag{2}$$

The adoption of the dyadic Green's function of a bianisotropic medium, for characterizing the superconducting microstrip structures of multilayered substrate and superstrate, necessitates the development of new permittivity and permeability functions in space domain as:

$$\varepsilon(x,y,z) = (\varepsilon_0 + j\frac{\sigma_0}{\omega})P_{W/2}(y)P_{t_0/2}(z) + (\varepsilon_0 + j\frac{\sigma_1}{\omega})P_{t_1/2}\left[z - \sum_{i\le n} h_i\right] + \sum_{i\le N}\varepsilon_0\varepsilon_{ri}(1 - jtg\delta_i)P_{h_i/2}\left[z - sgn(z)\left(h_i/2 + \sum_{j<i} h_j\right)\right]$$

with $$N = \frac{1}{2}\left[n(1 - sgn(z)) + n_s(1 + sgn(z))\right]$$

where n is the number of substrate layers, n_s is the number of superstrate layers and m is the number of lamina composite ground plane. We define the functions $P_\tau(x)$ and sgn(x) as follows:

$$P_\tau(x) = \begin{cases} 1 & \text{for} \quad -\tau/2 \le x \le \tau/2 \\ 0 & \text{otherwise} \end{cases}$$

$$sgn(x) = \begin{cases} 1 & \text{for} \quad x > 0 \\ -1 & \text{for} \quad x < 0 \end{cases}$$

The magnetic anisotropy of microstrip dielectrics is presented here by a permeability function similar to that of the permittivity previously described.

We note here that our assumption of the multilayered microstrip line as a bianisotropic planar medium requires, to characterize it, an infinitely long bianisotropic microstrip line of width W, with a longitudinal distribution current of the form:

$$f(x) = e^{jk_s x} \tag{3}$$

The analytical integration of electrical field integral equation with respect to x, gives the complex propagation constant k_s. The superconducting strip and ground plane are characterized by their thicknesses and complex conductivity using the two-fluid conductivity model (London model) [3]:

$$\sigma(T) = \sigma_n\left(\frac{T}{T_c}\right)^4 - j\frac{1}{\omega\mu_0\lambda_L^2} \tag{4}$$

where σ_n is the normal state conductivity at the closest value of temperature greater than the critical temperature T_c, λ_L is the penetration depth of the magnetic field in the superconductor called London length expressed as follows:

$$\lambda_L(T) = \frac{\lambda_0}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}} \tag{5}$$

with λ_0 is the penetration depth at T=0K.

The supposition that the strip conductor is not perfect ($\sigma\neq\infty$) necessitates the addition of and additional term in the electrical field integral equation for all the points in the strip. This term presents the losses in the linear approach.

Imposing the boundary condition along the strip width by integrating along y and z for all point in the longitudinal direction on the strip (x-direction), the electrical field element may be expressed as follows:

$$\vec{E}(y,z) = \int_0^{t_0}\int_{-W/2}^{W/2} \overline{\overline{G}}(y,z,k_s/y_0,z_0)\cdot\vec{J}_s(y_0,z_0)dy_0dz_0 = \frac{1}{\sigma}\vec{J}_s(y,z) \tag{6}$$

In this study, we expend the current density as:

$$\vec{J}_s(y,z) = \sum_{i=1}^{N_p} I_{iy}\, g_i(y) g_i(z)\vec{j} + \sum_{i=1}^{N_p} I_{iz}\, g_i(y) g_i(z)\vec{k} \tag{7}$$

and the υ-dependent basis for the current density $g_i(\upsilon)$ is chosen as a piecewise sinusoidal function given in [7,8]. N_p is the number of piecewise sinusoidal modes.

By weighting both sides of the integral equation (6) by an arbitrary vector function $\vec{w}(y,z)$ and then integrating over y and z, one can get a homogeneous integral equation given by:

$$\int_0^{t_0}\int_{W/2}^{W/2}\vec{w}(y,z)\cdot\left[\int_{-W/2}^{W/2}\int_0^{+\infty}\overline{\overline{G}}(y,z,k_s/y_0,z_0)\cdot\vec{J}_s(y_0,z_0)dy_0dz_0 - \frac{1}{\sigma}\vec{J}_s(y,z)\right]dydz = 0 \tag{8}$$

By using the spectral domain formulation of the dyadic Green's function defined in [5] and applying the Galerkin's procedure, where the bases of $\vec{w}(y,z)$ are the same as those of $\vec{J}_s(y_0,z_0)$, we can integrate analytically with respect to z- and k_z-dependents. Then, we obtain the following characteristic matrix equation [10]:

$$\sum_{i=1}^{N_p}\sum_{j=1}^{N_p} I_i Z_{ij}(k_s) = 0 \tag{9}$$

2.2. Sensitivity study

There are two methods for carrying out sensitivity analysis of a structure. The first one is the finite difference method. The second method is the adjoint network approach [1]. In addition to these two techniques, a direct method can be used if the structure performance parameters are expressed directly in terms of component parameters. We have used, in this work, the first approach.

To obtain the geometrical sensitivity of the microstrip line with respect to an arbitrary geometrical parameter ξ, it is necessary to extract a new integral equation of the electrical field for ξ. This integral equation is obtained by applying the definition of the total derivative, to the integral equation expressed by (1). Then,

$$\frac{d\vec{E}(x,y,z,\xi)}{d\xi} = \frac{\partial \vec{E}}{\partial \xi} + \left(\frac{d\vec{r}}{d\xi} \cdot \vec{\nabla} \right) \vec{E} \tag{10}$$

The integral equation of the electrical field given by the equation (1) can be expressed as follows [10]:

$$\frac{d\vec{E}(x,y,z)}{d\xi} = \int_V \left\{ \left[\left(\frac{d\vec{r}}{d\xi} \cdot \vec{\nabla} \right) \overline{\overline{G}}(\vec{r}/\vec{r}_0) + \left(\frac{d\vec{r}_0}{d\xi} \cdot \vec{\nabla}_0 \right) \overline{\overline{G}}(\vec{r}/\vec{r}_0) + \left(\vec{\nabla}_0 \cdot \frac{d\vec{r}_0}{d\xi} \right) \overline{\overline{G}}(\vec{r}/\vec{r}_0) \right] \cdot \vec{J}(\vec{r}_0) + \overline{\overline{G}}(\vec{r}/\vec{r}_0) \cdot \left[\frac{\partial \vec{J}(\vec{r}_0)}{\partial \xi} + \left(\frac{d\vec{r}_0}{d\xi} \cdot \vec{\nabla}_0 \right) \vec{J}(\vec{r}_0) \right] \right\} dV_0 \tag{11}$$

Basing on the fact that $\vec{\nabla}\overline{\overline{G}}(\vec{r}/\vec{r}_0) = -\vec{\nabla}_0 \overline{\overline{G}}(\vec{r}/\vec{r}_0)$, we can express the total derivative of the electrical field integral equation as follows:

$$\frac{d\vec{E}(x,y,z)}{d\xi} = \int_V \left\{ \left(\vec{\nabla}_0 \cdot \left[\left(\frac{d\vec{r}_0}{d\xi} - \frac{d\vec{r}}{d\xi} \right) \overline{\overline{G}}(\vec{r}/\vec{r}_0) \right] \right) \vec{J}(\vec{r}_0) + \overline{\overline{G}}(\vec{r}/\vec{r}_0) \cdot \frac{d\vec{J}(\vec{r}_0)}{d\xi} \right\} dV_0 \tag{12}$$

Applying the condition to the limit near the strip for the electrical field gives:

$$\int_V \left\{ \left(\vec{\nabla}_0 \cdot \left[\left(\frac{d\vec{r}_0}{d\xi} - \frac{d\vec{r}}{d\xi} \right) \overline{\overline{G}}(\vec{r}/\vec{r}_0) \right] \right) \vec{J}(\vec{r}_0) + \overline{\overline{G}}(\vec{r}/\vec{r}_0) \cdot \frac{d\vec{J}(\vec{r}_0)}{d\xi} \right\} dV_0 - \frac{1}{\sigma_0} \frac{d\vec{J}(\vec{r})}{d\xi} = \vec{0} \tag{13}$$

Note that the y-dependence and z-dependence of the total geometrical derivative of the current density $(d\vec{J}/d\xi)$ are expended over the same basis functions of the current density given by (7). We can write then:

$$\frac{d\vec{J}_s(y,z)}{d\xi} = \sum_{i=1}^{N_p} I'_{iy} g_i(y) g_i(z)\vec{j} + \sum_{i=1}^{N_p} I'_{iz} g_i(y) g_i(z)\vec{k} \tag{14}$$

After substitution, testing it with the same test functions $\vec{w}(y,z)$ and integrating over x, y and z, we obtain:

$$\int_z\int_y \vec{w}(y,z)\cdot\int_V\left(\vec{\nabla}_0\cdot\left[\left(\frac{d\vec{r}_0}{d\xi}-\frac{d\vec{r}}{d\xi}\right)\overline{\overline{G}}(\vec{r}/\vec{r}_0)\right]\right)\cdot\vec{J}(\vec{r}_0)dV_0 dydz$$
$$+\int_z\int_y \vec{w}(y,z)\cdot\left[\int_V \overline{\overline{G}}(\vec{r}/\vec{r}_0)\cdot\frac{d\vec{J}(\vec{r}_0)}{d\xi}dV_0 - \frac{1}{\sigma_0}\frac{d\vec{J}(\vec{r})}{d\xi}\right]dydz = 0 \tag{15}$$

Substituting of $\vec{w}(y,z)$, $\vec{J}(\vec{r})$ and $\frac{d\vec{J}(\vec{r})}{d\xi}$, and eliminating the term $e^{jk_s x}$ in the tow members of (15) as we have proceeded in (8), we permit to posing:

$$I_i W_{ij}\left(\frac{\partial k_s}{\partial \xi}, k_s\right) = \int_z\int_y g_j(y) g_j(z)(\vec{i}+\vec{j}+\vec{k})\cdot$$
$$\int_V I_i g_i(y) g_i(z)\left(\vec{\nabla}_0\cdot\left[\left(\frac{d\vec{r}_0}{d\xi}-\frac{d\vec{r}}{d\xi}\right)\overline{\overline{G}}(\vec{r}/\vec{r}_0)\right]\right)\cdot(\vec{i}+\vec{j}+\vec{k})dV_0 dydz \tag{16}$$

$$I'_i Z_{ij}(k_s) = \int_z\int_y g_j(y) g_j(z)(\vec{i}+\vec{j}+\vec{k})\cdot$$
$$\left[\int_V I'_i g_i(y) g_i(z)\overline{\overline{G}}(\vec{r}/\vec{r}_0)\cdot(\vec{i}+\vec{j}+\vec{k})dV_0 - \frac{1}{\sigma_0} I'_j g_j(y) g_j(z)(\vec{i}+\vec{j}+\vec{k})\right]dydz \tag{17}$$

Substituting (16) and (17) in (15) gives a system of linear equations:

$$\sum_{i=1}^{N_p}\sum_{j=1}^{N_p}\left[Z_{ij}(k_s) I'_i + W_{ij}\left(\frac{\partial k_s}{\partial \xi}, k_s\right) I_i\right] = 0 \tag{18}$$

The use of the obtained results, concerning I_i and k_s, by solving the homogenous matrix equation (9), allows the resolution of the system of linear equations (18). Hence, we found the geometrical derivative of the propagation constant. Therefrom, we can extract the sensitivity of the microstrip line with respect to its geometrical parameters by applying the definition of the sensitivity given by [11]: $(\partial\beta/\partial\xi) = -\mathrm{Real}(\partial k_s/\partial\xi)$.

3. Results

First, we calculate the effective dielectric constant, as shown in figure 2, for a microstrip line of GaAs substrate of ε_{r1}=12.6, h_1=0.1mm, and an isotropic dielectric superstrate of ε_{rs1}=1.0, and W=0.07mm for strip thickness t_0=0.3μm, versus frequency at T=77K.

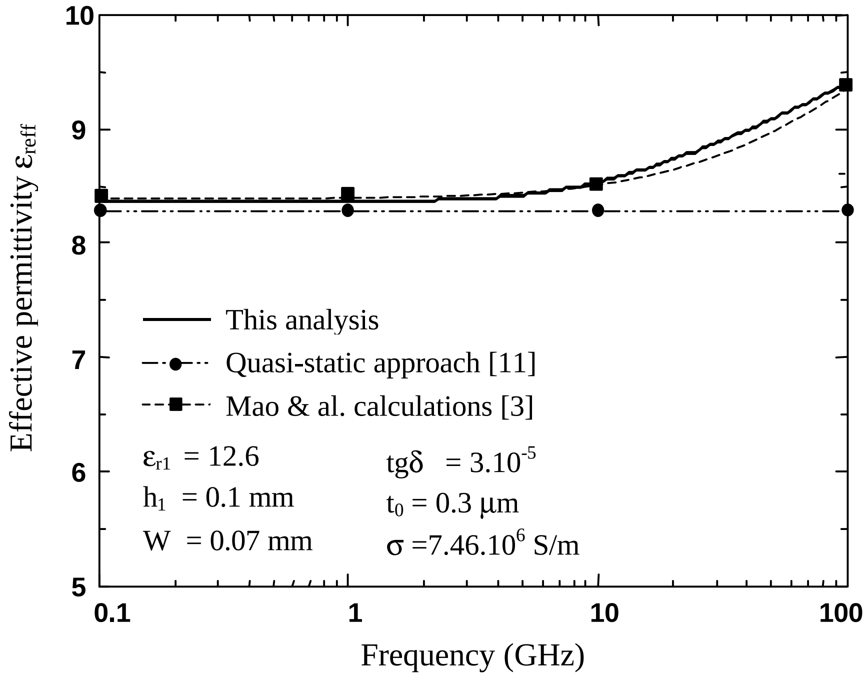

Fig. 2. Effective permittivity for microstrip line, of ε_{r1}= 12.6 substrate, h_1=0.1mm, t_1=12μm, t_0=0.3μm and W=0.07mm, versus frequency at T=77K.

The obtained results are in good agreement with those obtained by the quasi-static approach [11] for frequencies less than 10 GHz. Beyond this frequency, the two approaches diverge slowly. However, the results of S. Mao & al. [3] are very close to our calculations. In addition, the MSDA results are greater than our results for frequencies less then 10GHz. Beyond this frequency, the MDSA results become less than ours do. We can note that the effective propagation constant decreases as strip thickness increases.

The attenuation constant for the same superconducting microstrip line of figure 2 has been calculated and plotted in figure 3. Our results are in good agreement with those of Mao & al.[3] when the maximum difference between the two approaches is 3dB. This distortion is essentially due to the influence of the superconductor nonlinearity, which has not been considered in the approach of Mao &al. [3].

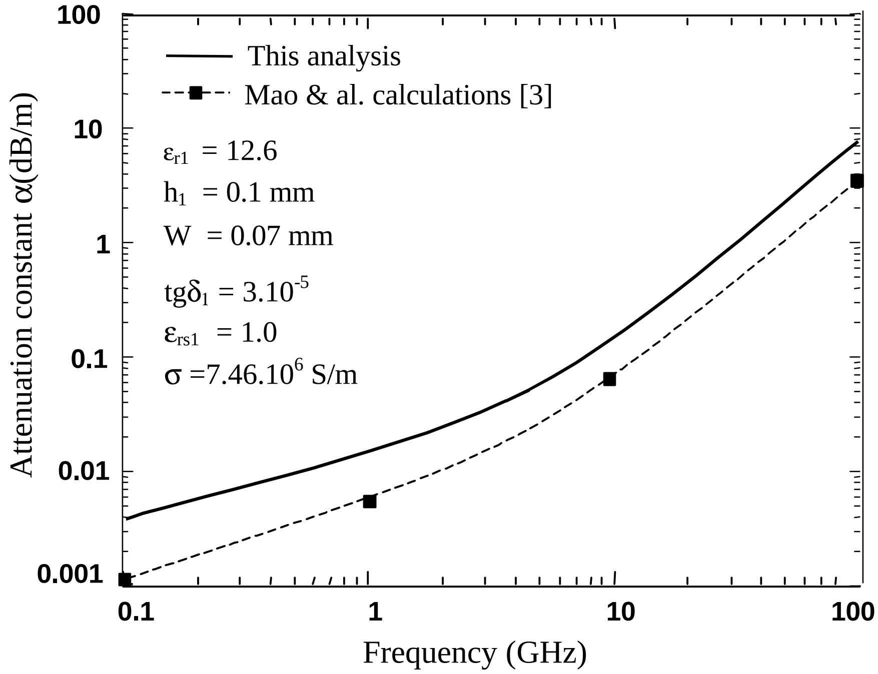

Fig. 3. Attenuation constant for microstrip line, on an ε_{r1}= 12.6 substrate, h_1=0.1mm, t_1=12µm, t_0=0.3µm and W=0.07mm, versus frequency at T=77K.

Finally, we calculate the effective permittivity sensitivity as function of the superconductor thickness, as shown in figure 4, for a microstrip line of GaAs substrate of ε_{r1}=12.6, h_1=0.1mm, and strip of W=0.07mm at T=77K and frequency f=10GHz.

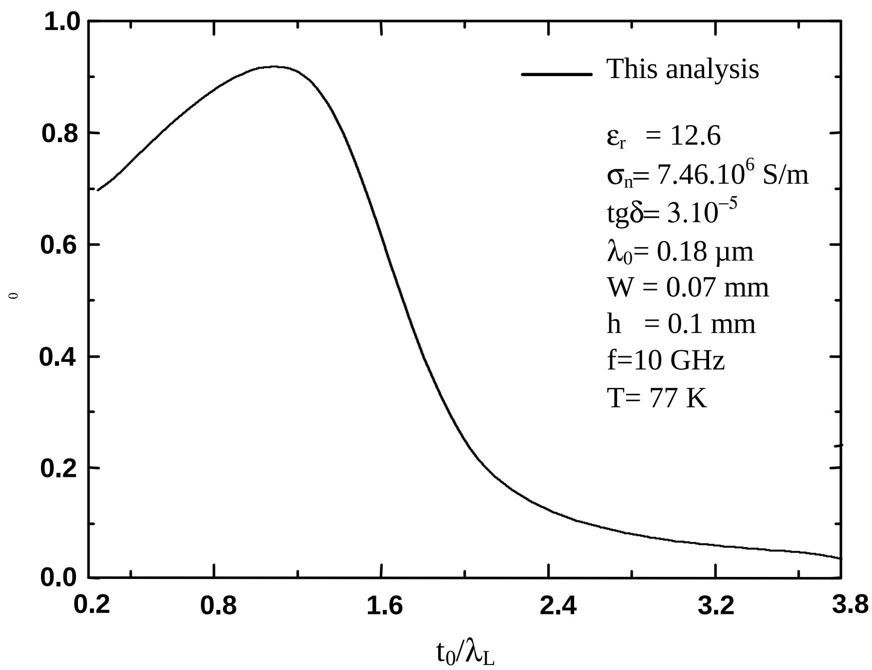

Fig. 4. Effective permittivity of superconducting microstrip of ε_{r1}= 12.6, h_1=0.1mm, t_1=12µm, t_0=0.3µm and W=0.07mm, versus Temperature at f=7GHz.

Our results show the highest sensitivity of the microstrip line to the thickness values close to the London depth. For thicknesses less than this depth as well as those superior to λ_L and inferior to $2\lambda_L$, the microstrip line becomes less sensible. For thicknesses greater than $2\lambda_L$, the microstrip line becomes insensitive.

4. Conclusion

We have developed a new full-wave theoretical model for study the geometrical sensitivities of the multilayer superconducting microstrip lines including the effects of superconductor nonlinearity. In this model, we have used the exact Green's function, of a bianisotropic medium adapted to a multilayered microstrip line, and the two-dimensional Galerkin's method. We have calculated the effective propagation constant and the attenuation constant versus frequency as well as the effective permittivity sensitivity as function to the microstrip thickness. The accuracy of our results has been checked with a good agreement with the different approaches.

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