# AN APPROACHING TO THE PROBLEM OF RECONSTRUCTION OF PHASE OF INTERFEROMETRIC IMAGES.

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**Abstract**

A spatial method of wave-front phase detection from an interferogram is presented. The algorithm used data- dependent system (DDS) methodology, in order to construct a statistical model named autoregressive model of movil averages.Its application in interferometry mainly aims to recover the function of coherence, to obtain the phase of the front of wave codified in the interferogram.

Keywords: data- dependent system, function of coherence, phase

## 1. INTRODUCTION

Many electronic instruments as computer chips, video recorders, optical instruments lens need a perfect finish. If the surfaces are too rough, the performance of such instruments can be very faulty; so that the finishing of such pieces needs the measuring of the surface texture. Toward 1980 it is relied on some methods of surface analysis, among them there is the interferometric technique, which uses the light ondulatory nature in order to produce a precise measurement of shapes and distances, that is to say, the topography of the surface. An advantage of the interferometric technique is that it doesn´t requires a physical contact with the piece of study. The mechanical methods of measurement fundamentally consist on a feeler or mechanic perfilometer provided with a diamond top wich lightly enters in contact with the surface and is slided through it, arising an output voltage proportional to the surface height in every contact point.

The general principle of the interferometric methods is based on the difference of optic paths ranged by a beam arised in a light source. (see Figure 1) , being splitted in a device (beam splitted), a part of this beam is reflected in a plane mirror and the other part falls over the surface of study, being recombined in the splitter.

The texture of the surface of study changes the optical track of the second face; when the faces is recombined they form an image of alternating brilliants and dark pixels, this image is the interference pattern or interferogram. The information about the spatial relationship in the beams that reflect the topography of the surface is contained in it. The spatial relationship between the interfering waves is measured by the phase angle; thus, a 0 radians phase means that the waves are reinforced between them. A phase of $2\pi$ radians corresponds to a wavelenght in the range of the beam, which mean the creation of an additional fringe. The inclination of the reference mirror creates a continuous variation of the road lenght of the beams path, as a result of this, a distribution of paralell interference fringes is generated, the texture of the surface alters this fringes pattern. The spacing of the fringes is determined by the surface and the inclination of the reference mirror. In general the interferometric technique uses three or more interference patterns which are obtained by starting from small displacements of the vertical position of the reference mirror or surface of the study. Starting from the measures of this interferograms the necessary information is obtained in order to get the phase between the surface and the reference for all the points. Starting from this information it can be inferred the topography of the surface which is finally is wanted one to measure. In the present work this technique are not used, but only an interference pattern starting from technicals and mathematical algorithms the phase are reconstructed.

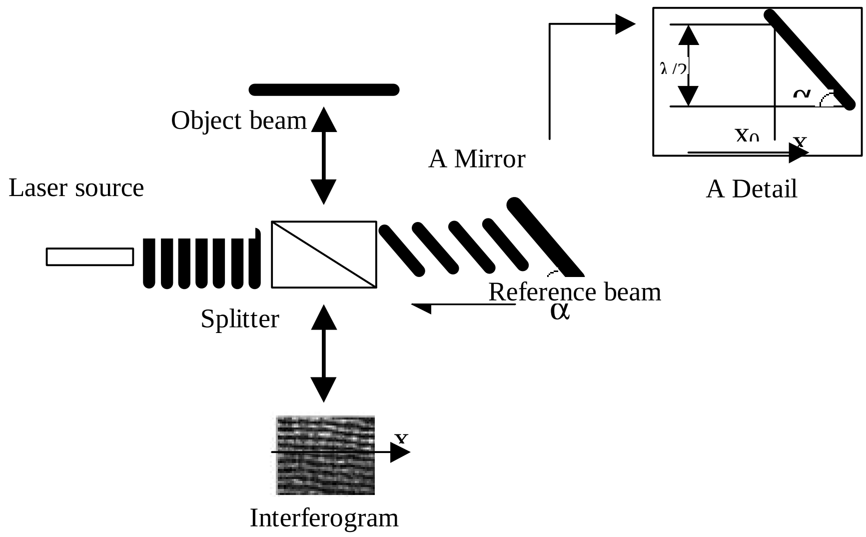


Figure 1. Interferometer schema

In short, the measuring of the surface state or surface finish of a piece by means of interferometric method requires the computing of the phase, which is present in a interferometric fringe pattern. The fringe pattern or interferogram is obtained by the superpositio of a reference face and an object beam in an interferometer. For very rough surfaces, as the majority used in engineering applications, it is used the oblicuous incidence interferometers [1,2,3]. The one used in this work is a version of the interferometer described in the [3] reference, and has been constructed in the LOTS laboratory. [4]. The fringe pattern registered in a CCD camera of 525x525 pixels, is digitalized to values between 0 and 255 gray levels, for a posteriory spatial scanning line by line in a normal direction to the fringes.

## 2. GENERAL OUTLINE OF INTERFEROGRAM ANALYSIS

The steps of the interferogram analysis are as it follows:

1. Digitalizing of the interferogram and elimination of the electronic noise.
2. Line to line softening in order to diminish the speckle effect (low filtrating passing).
3. Computing of the parameter values of the interferogram (continuous bottom, visibility of the fringes, function of coherence)
4. Calculation of phase value in each pixel.
5. Demodulación of the phase.
6. Unwrapping of the phase for every point of the interferogram.

This methodology is utilized for the various proposals to reconstruct the phase

## 3. INTERFEROGRAM ANÁLISIS AND PHASE RECONSTRUCTION

The rays emerging from a laser source (see fig 1) are splitted in two components by means of a beam splitter; a part is guided toward the surface and the other one is guided to the mirror. Both beams are combined in the splitter again. The reference surface is inclined at an angle $\alpha$. The resultant face contains the information of the mirror inclination and the deformation of the wavefront due to the surface studied. The spatial-temporal ratio is the essential feature which permits to carry out the temporal information (time of range of the optical paths), information which is coded in the luminous intensity of each pixel to the spatial information in the value of phase. For example, in the unidimensional case the difference of optical paths h is a function of the x position in the interferogram plane, this dependence is coded in the intensity values I, the analitical expression for I is:

$$I(x) = 2I_0[h(x)]\left(1 + g\left[\frac{2h(x)}{c}\right]\cos\left\{2p\left(\frac{2h_m}{l}\right) + f_s\left[\frac{2h_s(x)}{c}\right]\right\}\right) \qquad (1)$$

being h(x) = $h_m(x) + h_s(x)$, $h_m(x)$ is the displacement caused by the inclination of the mirror in the reference beam, such a displacement varies in a lineal form respect to x, and $h_s(x)$ is the shifting due to the roughness of the surface, this variation is generally randomic. The expression that relates phase $\phi$ with the shifting h are given by:

$$h(x) = \frac{l\, f(x)}{4p} \qquad (2)$$

The linear variation of displacement $h_m$ (x) can be written as:

$$h(x) = \frac{l\, f(x)}{4p} = \tan(a)\mathrm{x} = \frac{l}{2X_0}x \Rightarrow f_m(\mathrm{x}) = \frac{2p}{\mathrm{X}_0}x = \mathrm{w}_0 x \qquad (3)$$

where $w_0 = 2\pi / X_0$ is the spatial angular frecuency and $X_0$ the spacial period . By using equation (3) it can be written the expression for the intensity I as

$$I(x) = 2I_0(x)\{1 + g(x)\cos(w_0 x + f_s(x))\} \qquad (4)$$

In general the distribution of intensity in a fringe interference system, with one of the wavefronts inclined, can be described in the image plane as

$$I(x, y) = S(x, y)\left\{A(x, y) + B(x, y)\cos\left[w_o x + f_s(x, y)\right]\right\} + N(x, y) \qquad (5)$$

Where S(x,y) represents the multiplicative noise (speckle) due to natural coherence of the image, while N(x,y) is the additive noise. If the analysis of the interferogram is line by line, the fringe system can be described by the equation

$$I(x) = A(x) + B(x)\cos\left[w_o x + f_s(x)\right] \qquad (6)$$

Expression similar to equation (4) where A(x) and B(x) are the continuous background and the contrast of fringes respectively. This amounts are functions that vary slowly compared with the cosin function. The amount $f_s(x)$ is the phase introduced in the interferogram by the specimen surface we try to measure.

**4. A SELF REGRESSIVE METHOD FOR THE RECONSTRUCTION OF PHASE (DDS)**

As it was mentioned above an interference pattern can be described by the expression

$I(x,y) = 2\, I_0 (x,y) + \Gamma(x,y) + \Gamma^*(x,y) + n(x,y)$ (7)

That can be also written in the unidimensional case as:

$I(x) = 2I_0\{1 + \Upsilon(x)\cos(\omega_0 x + \phi_x)\}$ o : $I_x = A_x + B_x \cos(\omega_0 x + \phi_x)$ (8)

Being $A_x = 2I_0$ , $B_x = 2I_0\,\Upsilon(x)$ $\omega_0$ fundamental frecuency, $\phi_x$ the phase.

The expression $I_0\,\Upsilon(x)\cos(\omega_0 x + \phi_x)$ corresponds to $\Gamma(x) + \Gamma^*(x)$ and $\Upsilon(x)$ the fringes visibility: magnitude normalized of the function of coherence $\Gamma(x)$.

The DDS technique is a methodology for derivating a statistical representation of the interferogram data. The method exploits the fact of the correlation between boundary pixels; that is to say to considerate the interferogram data as a time serie, that means compute $I_x$ in terms of previous values:

$$I_x = q_1 I_{x-1} + q_2 I_{x-2} + \ldots + q_n I_{x-n} + a_x \qquad (9)$$

the $a_x$ form a secuence of randomic independent varables with mean 0 and variance $\sigma^2$ named residuals or remainders.

The expression (9) in the terminology of signals theory is known as a filter of response-impulse of finite duration (FIR) or in statistical terms, to considerate the data image as a time serie, experienced as a collection of observations in the time, in this case the observations given with respect to x position are available: I=I(x) or I=I(x,y), being (x,y) the position in the interferogram plane; that is to say, the data is represented by means of an autoregressive model AR(n).

The AR(n) process is choosen from a more general model called autoregressive movil model average ARMA(m,n), since it is very advantageous from the computational source of view. [8]. By analogy with the differential equations, the $\theta_i$ are the parameters of the characteristic polinomial of autoregression, supposing that equation (9) is a n-order differences equation, whose solution is a linear combination of functions in the form: $c_1\, l_1^n + c_2\, l_2^n + \ldots + c_n\, l_n^n$, where values $\lambda$ are solutions of polinomial:

$$l^n - q_1 l^{n-1} - q_2 l^{n-2} - \dots - q_n = 0 \qquad (10)$$

In order to evaluate the polinomial (10) it is necessary to know the θi parameters, for this there are several proposals, in this work is utilized the least-square method, that is to say, we minimize the expression

$$\sum_{x=1}^{m} (I_x - q_1 I_{x-1} - q_2 I_{x-2} - \dots - q_n I_{x-n})^2 = \sum_{x=1}^{m} (a_x)^2 \qquad (11)$$

Expression (11) corresponds to a functional F depending of θi parameters, deriving with respect such to parameters and equaling to zero the respectives partial derivatives a linear equations system is reached:

$$(A^T A)q = A^T B \qquad (12)$$

Where the A matrix, the B vector and the θ vector has the form

$$A = \begin{bmatrix} I_n & I_{n-1} & \dots & I_1 \\ I_{n+1} & I_n & \dots & I_2 \\ \dots & & & \dots \\ \dots & & & \dots \\ I_{m-1} & & & I_{m-n} \end{bmatrix} \qquad B = \begin{bmatrix} I_{n+1} \\ I_{n+2} \\ . \\ . \\ I_m \end{bmatrix} \qquad q = \begin{bmatrix} q_1 \\ q_2 \\ . \\ . \\ q_n \end{bmatrix}$$

In many cases the system (12) is “bad conditioned”, because of what it is necessary to apply any regularization process.

Once stimated the parameters θ, λ roots of (10) are found. In Pandit proposal [6,7,8], the autoregressive model upsets the interferogram data in different wavelenghts modes or periodicities corresponding to the roots, these components are stimated and their contributions to the signal variance are calculated.

$$g_0 = d_1 + d_2 + \dots + d_n \qquad (13)$$

Each $d_i$ depends of the significance or contribution of its respective root $\lambda_i$ , whose weight g are given by the expression:

$$g_i = \frac{l_i^{n-1}}{(l_i - l_1)\dots(l_i - l_{i-1})(l_i - l_{i+1})\dots(l_i - l_n)} \qquad (14)$$

and

$$d_i = \left(\frac{g_i g_1}{1- l_i\, l_1} + \frac{g_i g_2}{1- l_i\, l_2} + \quad . \ . \ . + \frac{g_i g_n}{1- l_{n1}}\right) s^{\ 2} \qquad (15)$$

Because the equation (9) is a n-order difference equation, we can write the $I_x$ expression by means of the convolution of Green's function (system impulse response) with the residual elements $a_x$ :

$$I_x = \sum_{j=0}^{x} G_j\, a_{x-j} \qquad (16)$$

The Green's function are given by the expression:

$$G_j = g_1\, l_1^j + g_2\, l_2^j + \ldots + g_n\, l_n^j \qquad (17)$$

Green's function indicates the way the system "remembers" the past stimuli. Since the dynamic systems point of view this function is known as impulse response. The expression (17) indicates the disaggregation in different ways corresponding to every root of characteristic polinomial.

By inserting (17) in the equation (16) we get:

$$I_x = \sum_{j=0}^{x} (g_1\, l_1^j + g_2\, l_2^j + \ldots + g_n\, l_n^j) a_{x-j} = \sum_{j=0}^{x} (g_1\, l_1^j + g_2\, l_2^j) a_{x-j} + \sum_{j=0}^{x} (g_3\, l_3^j + \ldots) a_{x-j} \qquad (18)$$

The roots of selfregression polinomial are ordered according to the significance of the ($g_i$) modes, that is to say: $\lambda_1$, : $\lambda_{2,}$ .... $\lambda_n$ such that: $d_1 = d_2 > d_3 >$ ....$d_n$. The two first roots represent the function of coherence.

According to the equation (7) and comparing with the equation (18) it we have:

$$\Gamma_x + \Gamma_x^{\ *} = \sum_{j=0}^{x} (g_1\, l_1^j + g_2\, l_2^j) a_{x-j} \qquad (19)$$

with $g_2\lambda_2^j = (g_1\lambda_1^j)^*$.

The third root represents the continuous background of the interferogram and in most of cases is a real root, that indicates a small exponential decline in the luminous intensity, in such away that we can to express the continuous background as:

$$2I_0 = \sum_{j=o}^{x} g_3\, l_3^{\,j} a_{x-j} \tag{20}$$

The other roots correspond to the noise present in the interferogram.

In summary we can write the formula (16) as follows:

$$I_x = \sum_{j=0}^{x} (g_1\, l_1^j + g_2\, l_2^j) a_{x-j} + \sum_{j=0}^{x} g_3\, l_3^j a_{x-j} + \ldots\text{noise} \tag{21}$$

In other words the Green's function G is decomposed in the following way: G= G' + G'' + G'''. G' represents the continuous background, G'' the function of coherence and G''' the noise. In short:

Continuous Background $2I_0 = \sum_{j=0}^{x} G'_J\, a_{x-j}$ (22)

with $G'_j = g_3\lambda_3^j$

Function of coherence and its conjugated: $\Gamma_x + \Gamma_x^* = \sum_{j=o}^{x} G''a_{x-j}$ (23)

with $G''_j = g_1\lambda_1^j + g_2\lambda_2^j$ .

Due to the linear independence of the equation solutions in differences (9), we get the expression for the function of coherence from the equation (23).

$$\Gamma_x = \sum_{j=o}^{x} g_1\, l_1^{\,j} a_{x-j} \tag{24}$$

Once calculated the function of coherence, the phase is the argument of such a function:

$$f(x) = Tg^{-1}\left[\frac{\mathrm{Im}(\Gamma(x))}{\mathrm{Re}(\Gamma(x))}\right] \tag{25}$$

In most methodologies of phase calculing as FTM (Fourier Transform Method) or DDS, the obtained phase (equation 25) is limited in its main value (see figure 2) that is in the interval [$-\pi$ , $\pi$ ], this is technically known as "wrapping phase", in order to find the true value of the phase some unwrapping algorithms must be applied, this process is known as "unwrapping phase". If we note the wrapping phase as $\theta_m$ and the unwrapping phase $\theta_n$ as the relationship between them is given by the expression $\phi_w = \phi_u + 2\pi k$, being k an integer, in other words the unwrapping process consists on aggregate (add or substract) a multiple integer of $2\pi$ to the phase value in each pixel, this process is done in each of the columns of the bidimensional map of phase $\phi_w$.

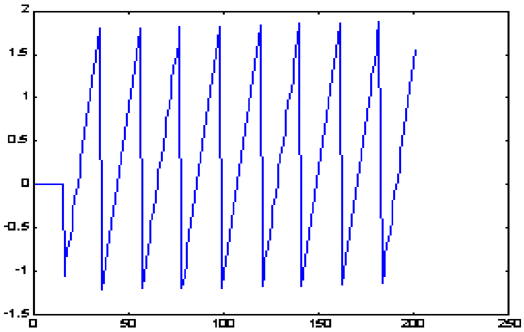


Figure 2. Phase wrapping (in radians)

## 5. RESULTS AND CONCLUSIONS

In this part the exposed methodology in the previous section for the interference pattern (figure 3a) will be applied. The interferogram analysis is unidimensional, we realize it the DDS process to every column. Then we select the order of the self-regression model AR(n), in this case we take n=15: AR(15). In other words, the equation (9) is:

$$I_x = q_1 I_{x-1} + q_2 I_{x-2} + \ldots + q_{15} I_{x-15} + a_x \quad \text{for x=16, ... m}$$

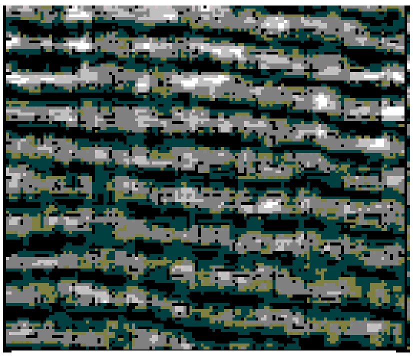

Figure 3ª. Interferogram

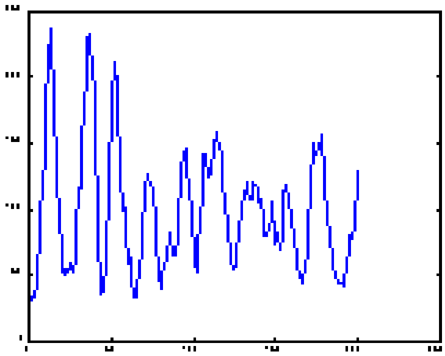

Figure 3b. Interferogram line

The parameters θ are stimated by using the equation (12):

$$(A^T A)q \quad = \quad A^T B$$

then, relying on the θ values the 15-order characteristic polinomial is solved.

$$l^{15} \; - q_1 l^{14} \; - q_2 l^{13} \; - \ldots \; - q_{15} \quad = \quad 0$$

As a result 14 complex roots and a one real root are given, then the reaminders $a_x$, Green's function, the dominant mode of the carrier frecuency $w_0$, are calculated. In the following graphic the remaining elements are given.

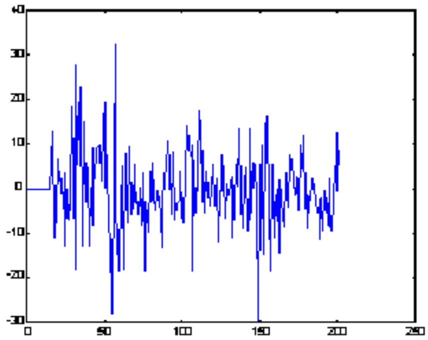


Figure 4. Residual elements $a_x$

The even dominant roots give us the value of carrier frecuency of wave $W_o$ that in this case is 0.30. Then we pass to discompose Green's function given by the equation (17) in the form G= G ´ + G ´´ + G´´´. Where G'' corresponds to the dominant mode (77.0326%) and from which we get the function of coherence with the equations (23) and (24) to obtain the wave phase further. The last step consists on substract the value of phase $\phi_u$ (unwrapped phase by means of some unwrapping algorithm) with the linear part of phase $w_0x$.

In figure 5a the unwrapping of the phase is shown. Then the linear part $w_0x$ is calculated. In figure 5b the difference between both phases is shown, which physically corresponds to the profile of a line of the surface in study.

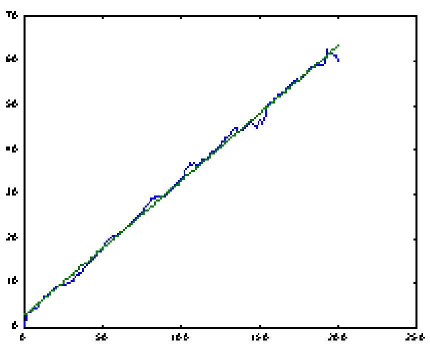

Figure 5a . Unwrapping of the phase and his linear part $w_0x$

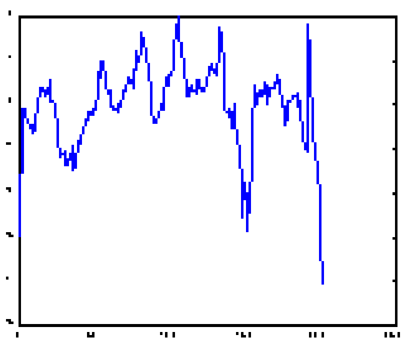

Figure 5b. Phase with the substraction of the linear part

By using the conversion phase equation (radians) to spatial measurements (nanometers)

$h(x) = \frac{l f(x)}{4p}$ we get an unidimensional approaching of the roughness in a scanning of the proof surface. In doing this process for every one of the lines (perpendicular to the fringes) of the interferogram, we would obtain similar graphics and with a high resolution grapher in 3D it would have a good approaching of the topography of the surface in study.

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