MANAGEMENT PROBLEMS IN UNCERTAINTY

MSNN'2000 (Mérida – Venezuela 22-24 October 2000)

Prof. Dr. Jaime Gil-Aluja
Universitat de Barcelona
Avda. Diagonal 690
08034 – Barcelona (Spain)
e-mail: jv00000@teleline.es

ABSTRACT

The problems that arise in businesses and institutions are currently immersed in a clear context of uncertainty. The solution to these problems is forever getting more difficult, if we are committed to the use of formal treatment valid for the sphere of certainty and in situations in which the required conditions are complied with for the use of probabilistic elements.

The theory of fuzzy sub-sets and their multiple variations, the development of which has been quite spectacular during latter decades, has certain characteristics that makes them perfectly adequate for these needs. For more than 25 years now we have been working on the creation and use of a methodology that is supported by this theory in order to arrive at an adequate treatment for economic-financial events in business reality. The result is, we believe, hopeful, since apart from providing theoretical solutions to specific problems, it has been possible to form a body of knowledge with a solid structure.

KEY WORDS

Fuzzyness, knowledge, management, uncertainty.

1. Introduction

The advance which at the time occurred with the revelation of techniques, today known under the generic denomination of "operational research", with its efficient application to solving, among others, complex problems arising out of World War II, led to a development, which we can qualify as "accelerated" in the study of theories and in the intensification of studies in this field. This research and discovery process was spurred on by the concern for its immediate application. A group of scientists and researchers formed a group around these ideas, thinking mainly on the possible solutions for management problems. And without any doubt whatsoever, due to this driving force and advances in technology, important objectives were attained in the order of forecasting, both within the sphere of economy and in management.

Today, many of those courageous writers, but above all their disciples, have embraced this new concept of tackling the complex realities of a society that is so changeable that it has become uncertain. The passage of a boolean logic to certain multivalent logic, in which nuances allow us to delve deeper into the essence of the problems themselves, has constituted an important advance for comprehension, explanation and treatment of the distorted mosaic of inter-relations into which the frenetic activity of those who have to manage the life of businesses and institutions has fallen.

As is well known, classical models relative to business decisions offer a simplified representation of reality. They cannot in any great detail bring to light the complexity of economic inter-changes, even at present in which a relatively large number of equations can be established and treated. Based on certain hypotheses the models select determined characteristic relations. The data for quantifying the same may be exact and certain. As we have pointed out, when this is random, resort can be made to the theory of probabilities.

But data is not always presented in a certain or random manner; in the majority of cases this data is uncertain. "Facts of life are uncertain; the economic social and financial surroundings of businesses is constantly changing; acts of men - because he is free and provided with an imagination - as well as relations between humans - because they are not robots-, are the profound causes of uncertainty"[1]

The works based on research by Boole, have allowed for enormous progress in the analysis of decisions and has facilitated the treatment of information by means of computers. Nevertheless, they are inspired in a logic that is too simple relative to current human reality: selection is not only done in terms of yes or no, of white or black. It is done in an area of greys.

What we can require from the models that we construct in order to represent both general and particular situations is that they be true to what we perceive. It is bad to deceive others but even worse to deceive oneself. And even more so for those seeking important answers. An involuntary and common way of deceiving is linked to semantic confusion, the sense we give or accept to give to words. In this way, some confuse subjectivity with objectivity, others uncertainty with chance. When speaking of chance, that is to say probability, and one is not capable of even vaguely justifying or specifying these probabilities, we are deceiving ourselves. It is absolutely certain that in all languages, in all dictionaries, chance is a synonym for uncertainty. Nevertheless, there is an essential difference between the contents of these two words: chance is linked to a measurement, that of probabilities, while, by definition, what is uncertain cannot be measured. Obviously the person who can measure and who should be measuring and does not do so, cannot pretend to be a researcher. But when this is not possible the resort remains for better knowing and expressing its subjectivity. Even

[1] Barre, R: Prologue to the work by Kaufmann, A and Gil-Aluja, J.: Técnicas operativas de gestión para

without the certain or probabilistic measurement, a certain scientific behaviour can be expected.

Under current circumstances, economists and businessmen are not relaxed, but they will be even less so in the future. The theories of yesterday have become as obsolete as the machinery on display in museums. In order to calm us down, because we all require them, new laws are proposed, rules that are closer to reality, certain supports for our mental relaxation, but those proposing the same frequently are deceiving themselves.

Facts that are susceptible to true repetition, belong to the sphere of nature, of physics, chemistry, astronomy and even biology, but man introduces, as well as those uncertain facts of nature, those that spring from his freedom and his power of imagination.

It seems to be an exaggeration in the pejorative meaning that is given to words such as subjective, uncertain, fuzzy, diffuse, just a small sample of a vocabulary of our day and age. By the very nature of things, by the thousands of paths along which scientific progress leads us, by the great variety of our thoughts, by the diversity of our opinions, it is much more difficult to make forecasts today than it was yesterday, and it is ever more complicated to place ourselves within the safety of certainty. The world, visualised both from a perspective of space and time, daily becomes more complicated because a great many unforeseen events appear that link together to form the web of our surroundings.

We have crossed the threshold of the "age of communications" and the information that scientific knowledge transports and makes known, is daily converting into combinatory all that we can capture and introduce as a basis for our decisions. Even with the aid of equipment for data processing, of whatever capacity, it is difficult for us to stay within the sphere of rationality, because the information and its propagation increases faster that the power of our equipment

---

el tratamiento de la incertidumbre. Publ. Hispano-Europea. Barcelona, 1987.

for treating it. And it would seem that this will continue for some time, unless the human being tires of inventing, becomes tired of possessing this marvellous curiosity which has led him to where he is at present and towards an exciting but .. uncertain future.

The idea of freedom is linked to adventure. Life itself is an adventure and particularly where imagination is present. The two concepts of freedom and security lead us directly to the concept of entropy. If there is too much disorder, life has no sense; when there is not enough it is no more than a mere procedure. We accept our surroundings with the uncertainty they contain and attempt, conveniently guiding our responsibilities, to attain sufficient success in accordance with our criteria, according to our deepest aspirations. But never let us reject adventure and the risks this implies, because we could convert ourselves into what we desire for our robots. Computers, are programmed beings, which, as they have been conceived cannot do without this state, but programming is not made for humans who, as such, should have the liberty to accept, of making mistakes, of winning, of loosing, but always carrying within the passion for creating new things.

This is not always easy. What is more, often it is difficult and even dangerous. There is an essential difference between a human being and a robot. The former is free, the latter is submitted to slavery. Even when talking about artificial intelligence for these robots, abusive use is made of the word intelligence, since, at least up till now, all it is, is adaptive programming. Between this and intelligence there is an enormous difference. This difference is imagination. This type of madness that belongs to us is what makes the difference.

Everything that is mechanical, that which can be programmed at present and in the future, must be done by machines, and the more efficient they are the more necessary will be imagination, adventure, ... uncertainty.

2. Science and change[2]

For some time now scholars of economy and company management are attempting to channel their activities towards resolving the serious problems that social, economic and business problems are creating as a consequence of the situation of uncertainty that is characteristic of our day and age. In this way, a large number of proposals have burst upon scientific spheres that, in different senses, converge to provide a new treatment both of old problems and of those that arise from the complex network of economic-financial relations. Each one of these proposals brings to light certain aspects that are intended to show the new streams along which research activity must flow. However, in many of them will be seen, under different guises, old routines that have been unable to generate horizons that are capable of breathing oxygen into the charged atmosphere arising from the comparison between likes.

Perhaps one of the reasons for so much enlightenment has been caused by the influence of the lengthy shadow of the "excluded middle principle", which has been a rule and guide for scientists for more than 2000 years. We feel that the moment has now come to state a new principle, that of "gradual simultaneity", which can include a large amount of logical reasoning that is capable of creating concepts, establishing methods and drawing up models and algorithms, which are at least suitable for providing some of the hoped for answers. In spite of this, we are conscious of the limitations that may appear and will go on appearing, as a consequence of certain works for which there is very little verified experience.

On numerous occasions we have repeated that scientific knowledge should not explain and treat the universe in which we

[2] The text of this section has been included from: “Elements for a theory of decision in uncertainty”. Kluwer Academic Publishers. London, Boston, Dordretch 1999, pages. 9-19.

would like to live but the one in which we really live[3]. For this it is necessary make an in depth revision of that knowledge that has been the "sacred cow" up to this point, based on certain laws that describe a stable world, and create a new structure of thought based on imbalances and unstable balances that lead to uncertainty, although to set aside these laws requires going back to ponder upon the suitability of traditional scientific reactions.

For many centuries one of the recurring ideas in western science was the idea of the laws of nature[4]. According to these nature follows certain rules structured around certainty. It would now seem to be the moment to bring to light the fact that, the contradiction of these schemes with the evolutionary aspect of the universe, in general, and the evolutionary aspect of the human being, in particular, is continually becoming more obvious.

All too frequently eyes are closed to say that certainties are believed in, even if they do not even reside in our world. Uncertainty must be accepted as a form of comprehending the rapid and successive imbalances that take place between the interactions arising from confrontation of objectives sought by social and economic agents. Basically, what we are doing is looking for a narrow door in the determinist routine that is alienating, since all therein is predetermined. Perhaps the result will be a new universe which even if uncertain, will respond to the normal rules of conduct of human beings in society, giving rise, even, to the enunciation of laws that can be formalised in a certain manner.

Obviously the sphere of management, always characterised by a tight network of interconnections, does not escape these considerations. But also it so happens that the subject of economic relations is constantly assailed by over-information that is difficult to

[3] Gil Aluja, J.: "La incertidumbre en la economía y gestión de empresas". Proceedings of the IV Congress of the Spanish Association on Technolgy and Fuzzy Logic. Blanes, September 14, 1994, pages 9-14.
[4] Gil Aluja, j.: "Investment in uncertainty". Kluwer Academic Publishers, Dordrecht, 1998, page 19-20.

assume and control, and also by certain profound changes in senses that are not predetermined, which make us look to a future surrounded by a veil of nebulous uncertainty. It would appear that the time of decline in stability and security has arrived. Faced with a world full of uncertainty, rigid specialisation no longer has any space, but is replaced with development of the imagination, creator of flexible and adaptive spirits.

On the other hand, the rapid evolution of social and economic phenomena makes the capacity of reaction to changes difficult, meaning that life for people is at constant boiling point. Above all it has been during the latter decades that changes have been greater, not only in socio-economic phenomena, but also in individual behaviour and ideas. Faced with this context, it would seem quite permissible to ask how is it possible to conceive a scientific activity when human thought, charged with a high degree of subjectivity, can attempt to seek the objective, amongst so many changes. The latent non-comformism in the more restless groups of the scientific community has made the inertia in the progress in economic knowledge give way to the birth of new ideas which, in very different directions, seek to give a reply to the numerous problems, which current society is disputing.

Determinist mathematics has been widely preached, and has prevailed, and even today continues to prevail, in many spheres of scientific activity in economics and management. But as important changes are taking place in society, which all the time are gaining greater presence in the economic field, certain voices are being raised clamouring for a new way of attacking problems. Thus the insufficiency of mechanicalism for explaining the phenomena and behaviour of the new society in an emergency is underlined. This radical change commenced after the Second World War. The figure of the "robot-man" is abandoned, time is considered as irreversible, formalisation of the fatalism of predestination is avoided, giving the

economic subject the opportunity to select its future freely, a future of which it is an active protagonist and not part of a cog in non-modifiable chain.

3. General considerations on decision making

Within the sphere of management, the concept of decision constitutes one of the most frequently used terms. Therefore, given the fact that economic systems are causing acceleration and deceleration processes that do not always pull in the same directions certain tensions of a different nature are caused in their midst that cause important and diverse problems. These problems are consequent upon the absence of a platform for the future with sufficient stability for establishing selection processes based on providing for schemes that at least allow the convenient delimitation of the occurrence of events.

In this environment, those responsible for companies and institutions have to take decisions with an economic and financial outcome that, as is well known, are not limited to the time at which they are taken but on many occasions are extended over several years. The difficulties of foreseeing and estimating, that are of an essence with every executive, are all the time increasing as a consequence of the recent climate of uncertainty.

When intending to carry out an adequate treatment of decisional problems, certain considerations[5] are convenient on the evolution of scientific thought, which have given rise to explanations of a different nature, and on many occasions contradictory, when there is a desire to explain why a substitution of certain theories for other occurs. In this sense, the repercussions of the confrontation between the positions of Popper[6] and Khun[7] have not yet died down.

[5] These ideas were expounded on in Gil Aluja J.: "Towards a new paradigm of investment selection in uncertainty". Fuzzy Sets and Systems. Vol. 84, number 2, December 9, 1996, page 187-197.

[6] Popper, Karl: "La lógica de la investigación científica". Ed. Tecnos, Madrid, 1971. The first edition of this work was dated 1934 and in it he inaugurates his philosophy on science, formulating the well known theory of the falsability.

We have no intention now to make a in depth analysis of this element that is fundamental for every researcher and will not event take sides in the debate that arose, except to point out, that in times such as the present in which society is undergoing profound changes, that during long periods research works done in the different spheres of knowledge a basic body of principles was accepted on which veritable formal buildings were constructed that have allowed for the development of social life.

Due to the outcome of events, on the one hand, and the new directions taken by research, the difficulty of supplying an adequate explanation to a large number of phenomena has been brought to light, all of which have been gathered into what has become known as "anomaly corner"[8]. These corners have become so large that they occupy an important part of many of the buildings constructed for housing the different portions of knowledge. Such an accumulation of anomalies has now become untenable for those who are attempting to provide an answer to the problems arising in the different levels of society. Sets of rules (or assumptions), that are the support for research works, and have been universally accepted within the different spheres of knowledge, are at first questioned and later substituted for others giving rise to changes in those processes that are susceptible to providing the solutions that the scientific community is clamouring for. In this way we sense a revolutionary process that constitutes the proof of the dissatisfaction of researchers relative to orthodox "official science", which gave rise to schemes that were initially considered as unorthodox.

In the sphere of management, attempts are being made, we feel that are only partially obtained, for creating certain elements that are

---

[7] Khun, Thomas: "La estructura de las revoluciones científicas". Ed. Fondo de Cultura Económica, Madrid, 1981. This work was written in 1962 and it is one of the many works written by the author on this subject.

[8] Gil Aluja, J.: Lances y desventuras del nuevo paradigma de la teoría de la decisión. Proceedings of the III Congress of the International Society For Management and Fuzzy Economy. Buenos Aires, November 10-13, 1996.

capable of arriving at an adequate treatment of the phenomena that are taking place within companies, when knowledge of the same takes place in such an imprecise manner that we are not even capable of numerically delimiting the schemes that are the result of the decision. Today we cannot, when faced with the impossibility of numerically including the complex and uncertain economic reality, resort to an initial simplification in order to carry out later developments based on these simplifying elements. Neither is it honestly possible to use uncertain numbers when the degree of uncertainty is such that they become useless.

To decide is to take sides for one alternative against another or others and management studies have developed, to a greater degree, with the search for elements that are capable of providing guidelines, in short to help those to whom the task falls, to pass from the draft stage to execution. In classical studies these aids on many occasions take the form of criteria based on concepts with a numerical support. Thus, present in practically all areas of economic knowledge are the notions of economic viability, profitability, productivity, ... which means information, always expressed in crisp numbers, that is contingent or uncertain. Now, to remain along this line requires asking the question as to whether it is possible to supply criteria when quite honestly it is not possible to establish certain numerical, nor even, subjective assignments.

Obviously we are very much aware of how to use numerical analysis, primarily in certainty and contingency, but also in uncertainty, above all during latter times.

4. The new principle of gradual simultaneity

The revision of the existing complex framework, accepted up to now by legions of university scholars who are in agreement with the convenience of what they have inherited, means that we must delve into the very principles that form the basis on which the technical elements that are required for handling the problems of decision are

constructed. From these arise four levels or strata: logic, pure mathematics, applied mathematics and management.

In the configuration of research activity, Western science throughout centuries has been constructed taking as one of its unquestionable supports, the excluded middle principle, from which, a logical architecture takes shape until it is consolidated in the works of George Boole. A single logical operator allows for the linking of proposals by means of two "modes": forward with the "modus ponens" and backward with the "modus tollens". The step to logical reasoning, expressed in words, to a presentation by means of symbols has given place in time to the development of mathematics which has acquired different "forms", but always within a mechanism, both when the path of certainty has been followed and when the path of contingency has been attacked. Based on these logical reasonings, expressed by means of symbols, specific techniques have been drawn up, nearly always in the form of models and algorithms, the objective of which was and continues to be the providing of instruments that are capable of rendering assistance at the companies and institutions. The use of these elements in the different spheres in which the management problem arises, constitutes the last step on this ladder that researchers have climbed in order to supply the necessary help to those having the responsibility of decision, and in this way avoid their falling into errors they are not responsible for.

At the present time and, as a consequence of repeatedly stated reasons and many others we could add, it is not possible to base all research activity on the excluded middle principle, which has become rather limited for housing all the "explanations" of the complex reality and phenomena that belong to our day and age. The formulation of a new principle is thus converted into a fundamental question if there is a real desire to open the door to the new adventures proposed by the future.

A few years ago we initiated the work of defining a principle hat was capable of harbouring the different logical operators that had been arising, which with great difficulty would have fitted within the excluded middle principle because their fitting in was only possible by artificial means. Our intention was also that the new principle could generalise the excluded middle. From these reflections arose the principle of gradual simultaneity[9]. This principle can be stated as follows: "A PROPOSAL CAN AT ONE AND THE SAME TIME BE TRUE AND FALSE, ON THE CONDITION THAT A DEGREE IS ASSIGNED TO ITS TRUTH AND A DEGREE TO ITS FLASENESS"

Let us move on, by means of certain examples, to bring to light the contents and meaning of this statement, at the same time as we show the relative position of the same with the traditional principle.

There are certain proposals for which it is normal for there to be no problems on accepting compliance of the excluded middle principle. Thus, when it is stated that "Peter belongs to the male sex", we conclude in the truth of this proposal and in the falseness of denying it. But other proposals can be made for which compliance in not quite so clear. In fact, proposals such as "Peter is tall" bring forward certain problems due to the relativity of the word "tall". In order to resolve the same, traditional studies habitually establish a threshold (obviously subjective and arbitrary) from which the truth of the proposal can be assumed. If in this case it is accepted that the threshold is 1m. 80 cm, then those that reach this height or exceed it will be considered as tall people, and those who do not reach this height will be assigned the falseness of the proposal. In this way we can admit that a person with a height of 1m. 80 cm is tall, whilst another person with a height of 1m 799 cm, is not tall. We feel that accepting the principle of gradual simultaneity allows for a good solution to this problem converting the excluded middle principle into a particular case. Let us now take a look at this.

[9] Gil Aluja, J.: Lances y desventuras del nuevo paradigma de la teoría de decisión. Proceedings of the

For greater ease we will continue with the same examples. With regard to the proposal on the height we will find those measurements for which the truth of the statement is fully complied with and also those for which the falseness of the statement is complied with. The tallest person in the world measures 2m. 38 cm and the shortest (negative proposal of the former statement) 0m. 56 cm. From these limits, in our case numerical, although there is no need for this to be so, we establish an order from the truth (tall) to falseness (short). This scale can be framed within an interval, for example [0, 1], or by any other concept that allows for creating an order. If, for effects of greater simplicity and, in order not to become separated as yet from the numerical sphere, the interval [0, 1] is accepted, we will assign a 1 to the truth, a 0 to the falseness, and therefore the giant at 2m. 38 cm will be tall in degree 1 and the dwarf of 0m. 56 cm, tall in degree 0. A person with a height of 1m. 70 cm could be considered as tall in a degree of 0.3 for example. In this way we establish that the nearer to the truth of the proposal, the assigned degree will be neared to 1 and as we move further away the assigned degree will be closer to 0.

But when we formulated the falseness of the proposal, by converting tall into not tall, or in other words short, what happens is that the same leading characters play a different role on being assigned a different degree. Thus, as short, the dwarf is assigned the highest degree in the value of 1, whilst the giant who is not at all short is assigned degree 0. The person then with a height of 1m. 70 cm was tall in a degree of 0.3 will now be short in a degree of 0.7, for example. In this way any given person is both tall and short at one and the same time, if we take the precaution of accompanying the qualifying word with a degree that can be numerically (as in our example) or non numerically expressed.

But to all those proposals, to which the excluded middle principle can be comfortably adapted, the principle of gradual

simultaneity is also applicable. What happens then is that of all the possible degrees only the extremes are assigned. In this way, with all reservations arising from the particular case, to the proposal "Peter belongs to the male sex" a 1 is assigned for the degree of truth and to the statement "Peter does not belong to the male sex" a degree 0 of truth. Therefore, he belongs to the male sex and does not belong, but in a different degree[10].

This brief exposition we hope, allows us to bring to light the generality of the principle of gradual simultaneity, that is capable of including a large variety of logical developments. In fact, faced with the limitations existing in formal logic arising from the precision of the excluded middle principle, the enormous flexibility and adaptability of the so-called multivalent logic has been observed. We will not expound any further on this subject but will, on the other hand make a reference to the passage of logic to fundamental mathematics.

5. Mathematics of uncertainty

When intelligent beings link words and/or proposals in a "reasoned" manner it is normal for them to find results. When these results are amalgamated together in diverse ways and always with the use of words, expressed either verbally or in writing, it is possible to get quite far in the linkage of ideas. All this fits into the sphere of logic. But both words and proposals are susceptible to being represented by means of certain symbols, in the same manner as the links between the same can be expressed by operators[11]. In this way it can be considered that we arrive much further in the objectives or results. When acting in this way on passing through the frontier of words and arriving at the sphere of symbols a passage is made from logic to mathematics.

[10] In all our examples on levels of degree in [0, 1]we have considered complementation in relation to truth - false.

[11] We have simplified this statement here even at the risk of falling into certain errors, with the object of not excessively breaking the thread of our reasoning.

For nearly 50 years now, many attempts have been made to carry out a formal construction of a type of mathematics starting out from incipient multivalent logic. Little by little the residues of these attempts were forming a medium from which an idea[12] arose that was capable, over time, of agglutinating hundreds of researchers whose works would be capable of opening up the way towards the mathematics of uncertainty. In the first place the development of the numerical elements, that paved the way to a new arithmetic of uncertainty, took place. Thus, faced with operators called "hard" that were typical of the mechanisms and suitable for handling objective problems, other operators were incorporated, considered "soft", which exercise a central function when trying to amalgamate, as well as possible, elements with a high charge of subjectivity. If the operator that is representative of the first of these cases is the composition of addition-product, then the operator with the greatest meaning for the second cases is the maxmin composition or convolution. During the latter third of the XXth century, it will be seen that mathematicians and engineers in the main, have been passing over from the filed of classical research to this new way of formalising their reasoning. Researchers, are striving in the analysis and development of new concepts, providing a system of symbols that is capable of identifying those elements that make up the merging mathematics, differentiating them from those that represent the linking of mechanist reasoning. Parallel to this, perhaps with a certain "gap" certain, concepts are opening up the way, nearly always arising from combinatory schemes, which allow us to visualise the formation of a nucleus of basic knowledge with a sufficiently unitary contents in order for us to talk about a non-numerical mathematics of uncertainty. In this way what is being achieved, all the time with greater frequency, is the expression with greater fidelity of multivalent logic by means of the numerical and non-numerical mathematics of uncertainty.

[12] We are referring to the work of Zahed, L.: "Fuzzy Sets", Information and Control, June 8, 1965, pages 338-353 from which arose the so-called Theory of fuzzy sub-sets.

All of us, whom in some way, find ourselves involved in social, economic or management research, are conscious of the fact of the need for "instruments" that are suitable for treating problems of decision and that, with difficulty can be supplied by mathematics in its purest state, at least immediately. From this stems the importance of an applied mathematics, which, in the different fields of knowledge may take different names. Let us recall in this respect operative research of classical studies or the works included under the name of operative management techniques in more recent times. The incorporation to scientific wealth of works carried out within the sphere of pure mathematics have facilitated (and continue to do so) the construction of tools that are destined to the treatment of the realities of our day and age. These tools are presented by means of models or algorithms, both supported by the solid knowledge that on many occasions takes the form of theories.

All we have just pointed out should not induce the reader to think that these are always new instruments that have appeared suddenly and instantaneously, but which are in fact, to a total or partial degree, re-formulations of known concepts or adaptations of schemes used with success at other times. It is certainly true to say that only with the existence of this part of mathematics, that is applied mathematics, would satisfactory solutions have been found in a context of uncertainty. However new concepts and new techniques are continually occupying more space, slowly relegating to a second level, all those that were previously in existence. We should consider that the so-called theory of forgotten effects[13] and the theory of affinities[14] are clear exponents of what we have just stated.

---

[13] Kaufmann, A and Gil Aluja, J.: Modelos para la investigación de efectos olvidados. Ed. Milladoiro. Santiago de Compostela, 1988.

[14] See for example, Kaufmann A. and Gil Aluja, J.: Técnicas especiales para la gestión de expertos. Ed. Milladoiro. Santiago de Compostela 1993, pages 151-175.

[15] Let us recall that the concept of valuation is similar to a numerical assignment made subjectively. Subjectivity id the characteristic that differentiates it from measurement, which is an eminently objective characteristic.

As a final point it is convenient to point out that the transformation of traditional models of a numerical nature to the field of uncertainty, based on the substitution of crisp numbers by uncertain numbers, generalises the same and even makes them more suitable for handling reality. Now, in itself, this transformation is not sufficient for us to tackle the complex reality of our day and age. It is the uncertain models, developed from concepts that emerged from non-numerical mathematics, that are capable of filling this void, which all the time becomes deeper, in the study of certain phenomena that escape, not only measurement but also valuation[15], even when this is done by means of the simplest of uncertain numbers.

The incorporation of these instruments means a clear break relative to preceding situations. And this, even when in their development, elements are used that have previously been used in classical studies. The current situation of equilibrium in the cohabitation of numerical models and uncertain non-numerical models is giving way to a greater supremacy of the latter, as a consequence of the growing difficulties even of being able to delimit social, economic and management phenomena. For this reason, those concepts that inevitably require to be numerically expressed (in certainty and in uncertainty) are no longer the pace setters due to the difficulties of expressing them objectively and even subjectively, taking into consideration the increasing uncertain context in which they are set down.

6. Management problems today

At this commencement of the millennium society is being faced with profound changes which neither the economic system nor business activity can avoid. Faced with such a changing reality and, as a consequence of the same, uncertain, businessmen are submerged in new problems the solution to which requires a large dose of imagination and, above all, a very solid preparation.

These new problems arise, on the one hand, as a consequence of the rapid evolution of the social surroundings in which businesses move and, on the other, by the continued and accelerated technological development that at frequent times modifies the economic horizon. All would indicate that this tendency is not only going to be maintained but that it will accentuate over time.

In this context, business executives must take decisions and adopt strategies the economic and financial repercussion of which is not limited to the time in which they are taken but extend, in many cases, over several periods. The difficulties of forecasting and estimating variables, that go hand in hand with the task of the executive, are all the time increasing more frequently as a consequence of a growing climate of uncertainty. The world of our predecessors evolved slowly, the current world and the world awaiting us in the very near future is immersed in a system of rapid inter-actions.

Traditional values, considered as the solid guide for many generations, have disappeared. Virtues such as hard work, perseverance, patience, previously extolled and admired, are today relegated to a second level, giving way to audacity, a competitive spirit, to the "kingdom" of the image, ...It was not too long ago that one had the necessary time for thinking and deciding. Today events happen so fast that they overtake us.

Economic life, in all of its possibilities, is submerged in this context and decisions that have to be taken within its realm are ever more complex as a consequence of the uncertainty in the outcome of future events. In this sense:

- Legal provisions are constantly changing the rules of the game.

- All the time external economic influences are modifying the expectations of the man of action.

- With ever increasing rapidity technology requires variations in production and distribution.

These and many other elements make management techniques change at a very high speed in order to adapt to the new configuration of the economic world.

Machines, which have always been the best support for human activity, are all the time carrying out greater and more varied tasks, freeing man from the more routine or more arduous jobs. In workshops automation has advanced as far as robots, in administration calculators have given way to modern computers. It is not difficult to think that in a not too far distant future the workman and executive will be limited to certain tasks for which imagination will be essential. Their powers will be COMPETENCE AND INITIATIVE.

It is not easy to change habits and carry out the necessary adaptation to this new world. For many years the word "specialisation" has been used. In the future this concept will give way to another: "adaptable qualification". For this PERMANENT TRAINING will become the norm. If specialisation becomes diluted, if machines substitute workers in routine and arduous tasks, the fate of the "proletariat" is its disappearance only to be substituted by the "robotariat".

Advances that are occurring and will occur in computers and in robots are simply enormous. Computers will become the irreplaceable companions of men of action, since power will only be exercised if supported by information. What will become important is to know how to filter and use it, because the information exists and all the time is becoming more and more abundant.

In order to be able to face these changes businesses are also modifying their productive structures and attempting to offer new products and new services, which in some cases are merely variations of those already existing, and others are truly new products. Just a

few years ago banks, for example, offered certain limited financial products; sight savings accounts, savings accounts at one year, long term savings accounts, current accounts, and a specific range of credits. These financial products have enjoyed a long life. Today the panorama has changed. At present new financial products are appearing by the day. Banks and Savings Banks are intensifying advertising of certain products that appear to be original. But as soon as they hit the market, another financial institution immediately appears with a launch of other products with the object of attracting eventual clients. This is only a very small example of the many that can be seen on a daily basis. Diversification is also a constant of our era.

What has occurred then is a change that constitutes a veritable revolution in the conception of business activity and for those who are researching in this field seeking new schemes that are capable of treating these new situations. The models that traditionally appear in texts on management with a foundation on certainty and probability, although they continue to be valid in many circumstances as we have repeatedly said, do not manage, in their current state, to reflect in a reliable manner this new world based on uncertainty.

It is for this reason that scholars of economic problems in business are seeking new techniques, that are suitable for including, in all their complexity, the phenomena that daily life raises, in order to be able to formalise and act on the same.

Nobody has any doubt that social groups, whatever their relative position, and whatever their geographical location, are faced with profound changes that are undoubtedly going to provoke upheavals, the consequences of which are difficult to foresee.

By the mere fact that in society, with the addition of elements with the capacity for coexistence, are causing certain processes of acceleration and deceleration that not always pull in the same direction, pressures of a diverse nature are taking place within its

core, that in some way, cause problems due to the lack of a platform for the future with sufficient stability in order to be able to make estimates on the variables which could conveniently limit the outcome of events.

Neither the economic system in general, nor business in particular, are immune from this new framework in which they are carrying out their activity. It is for this reason that their vitality and even survival will depend on their capacity for adapting.

But to adapt to new phenomena means overcoming a host of problems in the solution to which we university professors, who are dedicated to teaching and research of matters related to business activity, cannot feel left out. We are going to have to face up to a changing reality which is going to require a new vision and a different manner of seeking the solution to the problems that crop up.

7. Measurement and valuation in management

Within management studies the use of a determined group of techniques for measurement take place, which seek, by following the concept of Galileo Galilei "To measure what is measurable and attempt to make measurable that which is not so". Nevertheless, not all that has an interest in the life of business economy can be quantified, but, only portions of the phenomena, facts and relations are actually susceptible to measurement (that is to say, to objective numerical assignment) and there is an important task to be carried out until such time as the quantifiable field can be extended to all aspects of management.

Now then, in management studies, this reality should not prevent consideration being given to these phenomena that constitute a problem and which are not susceptible to measurement and incorporation to operating techniques that are known as classical; and this is so, because over the last few years new treatment possibilities have appeared that avoid measurement by means of subjective

estimates, and which are capable of being expressed numerically through what are now known as "valuations".

There is no doubt whatsoever that reality is precise in itself, but in the process of its conception a more or less large part of its nuances are lost: the human brain sees its surroundings in a simplified manner. And this is so because anything that is examined, however small it may be, and excusing the example of the atom, is so complex that getting to know it can only be done by means of a simplifying process. Even when looking at an object, our eyes see it in a gross and schematic fashion, which will become sharper as we use more and more powerful lenses. But in reality there is no microscope whatsoever that is capable of showing it "just as it is".

The fact that formalisation normally signifies restricted vision, obliges the researcher to select a path between carrying out from the very start a selection of elements to be considered, in order later on, to operate with precise instruments, or capture the reality in all its imprecision and operate with this "fuzzy" data, even knowing that the results will be given in an imprecise manner. The decision is reduced to selecting between a precise model, but a model in fact that shows reality imperfectly and a vague model that is more appropriate to reality.

If to this we add the fact that both human thought and actions are the result of a mixture of intuitions and strict logic, their study, on being carried out by means of certain simplified schemes, cannot be at the same time representative of reality and absolutely precise in their quantification. This has fed a tendency to resist change in the use of mathematics for studying the phenomena of human sciences. All too frequently it is considered that mathematics are a useless "complication" to facts and relations that in reality are quite simple.

In this context it is frequently sustained that only knowledge, which is susceptible to "measurement", can be considered as scientific. If this were so, what a weak role social sciences would fulfil.

All the more frequently there are fewer situations in which precision allows for measurements in certainty and events are sufficiently well known or repetitive so that the notion of chance is associated to an idea of measurement by means of probability. With greater frequency uncertainty crops up, which is taken in a subjective manner associating it to improbable facts, as the concept of sensation and the notion of valuation, both with a fundamentally subjective nature.

Uncertainty is consubstantial with human thought and prevails over many of its reasonings. In the attempts to formalise social behaviour in general and that belonging to the business area in particular, it becomes increasingly necessary to introduce this reality, even at the expense of having to do without the objective concept of measurement. When a phenomenon is not measurable a valuation, comparison, grading, relation, etc. must be made, which already constitutes, in itself, an important activity for the human brain. This is the reason why the incorporation of these subjective concepts signifies a significant advance towards a better knowledge of the phenomena.

If a situation cannot be defined exactly, but it can be stated that it is better than another, at this point there is a passage to a higher state of knowledge. But when it is said that in the future an event is more "possible" than another, a fundamental field is being opened up in the perspectives for reason and decision, since with this, subjective knowledge can be submitted, practically, to all the mechanisms of logic. Because here we have what is absolutely basic for the distinction between the lack of precision and the lack of exactitude. What is imprecise and what is fuzzy does not necessarily have to be inexact. In formal logic a thing is either true or false, but it cannot be the two things at one and the same time, as no nuances can be accepted, while studies of fuzziness assign a fundamental importance to the "degree" or level of the reality. Human thought is, in itself, full of nuances, full of vagueness.

8. Randomness and uncertainty

We have already stated that uncertainty and randomness are words that are used frequently and indistinctly in the scientific sphere. Nevertheless, no man of science will forget the fact that there exists a differentiation between what is measurable and what is not measurable. Obviously, in daily language, for daily communication, this confusion is accepted without it creating too many problems. But, when thinking about how to treat problems by means of scientific methodology, then the precaution must be taken to make this differentiation. Mathematics for treating uncertainty and mathematics for treating randomness are not the same[16] as it is necessary to distinguish one from the other at the same time as knowing how to associate them correctly when this is useful or necessary.

The main mathematical tool for treating uncertainty is the theory of fuzziness and valuation with its many variations, while the theory relative to randomness is that of probabilities. Right up to a certain time, the former was little know or unknown both by pure mathematicians as by specialists in applied mathematics. A hundred years ago the same occurred with the latter. Fortunately, a few years back there was an epistemological change and today there is greater knowledge on how to separate and associate what is measurable and what is not, when this is necessary.

It can be stated that, with the object of maintaining survival and to reproduce, every living being has carried out and is carrying out strategies, in an attempt to overcome extinction of the species, sometimes maintaining the same state, other through a process of transformation. Every living thing is in a permanent state of learning, of some cases over millions of years, in others instantly. Thus instinct would be the result of a lengthy learning period and intelligence the

[16] Kaufmann A. and Gil Aluja, J.: Las matemáticas del azar y la incertidumbre. Publ. Ceura, Madrid 1990.

result of a short one. But what is true of both, in any event, is the learning.

Man, as in all living beings, evolves in an uncertain sphere. One of the objectives of all species is to reduce the effects of uncertainty. And for this it is necessary to collate repetitions, take them into consideration and make use of them in order to decide.

The transition from uncertainty to randomness and from randomness to decision is a procedure the study of which could be particularly interesting. In the first place what is the difference between these two words: uncertainty and randomness. For may people, among whom can be found evolved and even intellectual people, these words mean very nearly the same abstract concept: not much or even nothing is known about what is being considered. Also, even the better dictionaries do not clearly define the difference. And yet uncertainty and randomness do not correspond to the same level of information. Uncertainty possess no laws, randomness does posses laws, either known or not, but that do exist by hypothesis. Uncertainty is deficiently structured and when explaining it this is done subjectively. Randomness, on the other hand, is tied to the concept of probability, which in itself is a measurement on observations repeated over time and/or in space; randomness is a measurement on observed facts, as it constitutes an evaluation which it is desired would be as objective as possible.

The frontier between what is objective and what is subjective is particularly vague. What is considered objective is that which is accepted without limitation by all or by a sufficiently important or powerful group. What is subjective is that referring to an individual or small group.

All too frequently it is stated that an event is probable when it should be said that it is possible. Probable is linked to measurement, possible with valuation, a somewhat weaker, softer concept, but that could be, on many occasions, more honest.

Therefore, words such as chance, randomness, stochastic should be used for those situations where laws of probability are available and accepted. On the contrary it would be necessary to use words such as uncertain, uncertainty, imprecision, fuzziness. To put this another way, situations in which measurements can be carried out and justified would not be confused with situations in which such measurements are no viable or cannot be accepted.

Randomness is structured uncertainty, measurable by means of logical and/or statistical reasoning. In olden times randomness was called στχοξ which represented the idea of an objective to be attained, of opportunity, of foretelling, of foreseeing. Nowadays the word "stochastic" is linked to every process in which its states can be given by means of probability. First the notion of probability was intuitive (works by Pascal, de Laplace) and then a strict axiomatic was introduced by Borel and another by Kolmogorov.

The preparation of a decision, either simple or complex, is an organisational activity of the mind in which intuition and logic are combined and where, frequently, it is difficult to make the separation. Mathematics often serves as an aid to the mechanisms of logic, although today we are also capable of using it for stimulating the imagination and sharpening intuition.

The models are supported, the majority of times, on determinist and probabilistic theories. Determinist models take into account certain data, probabilistic models data that is strictly measurable or constructed from reasoning that allows for accepting a priori the laws of probability. The novelty appears with the fact of knowing how to construct models in the event that neither the laws of probability nor the reasonings that are related to them can be used.

Why does a greater interest for the treatment of uncertainty occur now? Because it is all the time becoming more difficult, in spite of the progress attained in processes and machines, to arrive at sequences of data that are sufficiently stable, stationary and even

significant. Phenomena relative to human sciences are at present all the time getting less foreseeable in a clear manner. On the other hand less formal mathematical instruments are available, although these are no less strict (fuzziness does not exclude strictness).

We should consider the fact that what is important in a model is, in the first place, its reliability. If what is real is only appreciable by means of sensations instead of events, would we not be wasting this information if we rejected these sensations? Also, an event in human sciences in nothing else but a consensus of sensations. And even objective science is only objective by the fact that it is understood by the great majority or the most powerful group. No law whatsoever exists that one day or the other or sooner or later, is not questioned. Another will be found which will contain the previous one as a special case or even will not contain it in which case it will be revolutionary. The scientist knows how to remain within modesty and treats in the best possible way, within the current state of knowledge, that which is within reach.

9. Introduction to fuzzyniess in management

Current reality, characterised by change and uncertainty, does not allow in the majority of cases, to take into consideration data from the past, even if immediate, in order to establish a forecast that is sufficiently valid relative to the future. And what is most important is that this tendency is not going to diminish, but it is foreseen that the process will be accentuated with an acceleration, which will give way to continuously changing situations.

This substantiation, which is valid for the most elemental facts of peaceful coexistence, is also valid for the phenomena of businesses and institutions. To stop to think about this reality inevitably means reconsidering the hypothesis on which models can be built, if it is desired to reach greater comprehension of the operation of the sub-systems that form a business and constitute a valid path towards decisions.

It is obvious that there exists a preference for the construction of those schemes that are based on precise and "secure" data. However, this can become inoperable when reality does not allow for arriving at the same. Therefore, when this information is not available it is necessary to resort to certain numerical estimates of a subjective nature that we call "valuations".

Following a reverse order of preference we can consider[17] five types of models relative to the different levels of information:

1. A non-determinist model with unknown states.
2. A non-determinist model with known states but with events that are not valuable (we know the possible states, but we do not know how to assign to them a subjective scale of values).
3. A non determinist model with known states and events that are valuable but not measurable (we know the possible states, we can assign to each event of each state a value, taking into account that this valuation is not a measurement).
4. A non-determinist model with known states and measurable events (we know the possible states and can assign a probability to their occurrence).
5. Determinist model (we know the states and we consider the hypothesis that the event of a specified state is known).

In management studies, a modelling process is carried out in an attempt to go from level 1 to level 5. But the economic situation, we have described, makes for investigations stop at lower levels. This is the reason why resort has been made to the more general of the theories that are capable of describing uncertain situations: the theory of fuzzy sub-sets.

It is very well known that the theory of fuzzy sub-sets is a part of mathematics that is perfectly adapted to the treatment of both what is subjective and what is uncertain. It is an attempt to include

[17] Kaufmann, A.: Models mathematiques pour la stimulation inventive. Publ. Albin Michel. Paris, 1979, Page 53.

phenomena just as they appear in real life and carry out their treatment without attempting to deform them in order to make them precise and certain.

The formalisation of uncertainty, commencing from fuzzy concepts, has given rise to a different manner of thinking that includes the precision of sequential reasoning and the richness of the imagination, associating the sequential possibilities of the machine with the possibilities of the human brain.

The use of fuzzy schemes takes place today in practically all fields of study. It can already be found in business management, in biology, in medicine, in geology, in sociology, in phonetics and even in music, just to mention a few areas. Every problem that is located in the sphere of uncertainty is susceptible to being treated by means of the theory of fuzzy sub-sets and their multiple variations, since as time goes by it becomes more feasible to introduce, into formal schemes, mechanisms of thought such as sensations and numerical opinions.

For more than 50 years now a large number of mathematicians have been interested in multivalent logic, among these we should mention RUSELL, LUKACIEWICZ, POST, etc., but it is in 1965 when LOFTI A. ZADEH publishes his first article on "Fuzzy Sets" and a further 10 years must pass before a certain expansion occurs, because up to 1975 just a mere handful of works on this subject had been published.

Today, it is not possible to speak of one single logic, because there can be as many logical developments as one may be inclined to imagine. For this it is sufficient to establish certain axioms so that, as from them, propositions are correctly organised so that contradictions are avoided.

Thus, today a "fuzzy logic" is conceived in the same way that there was no problem at the time in conceiving a boolean logic. What

is more, if for the relations of man to computer in the current situation it is essential to resort to binary logic, for the relation of man to his fellow men it would appear to be more suitable to use the theory of fuzzy sub-sets.

The traditional theory of sets and boolean algebra, with its membership or non membership logic, has allowed for the formalisation of certain situations that occur in reality, but others exist that are difficult to model by means of these same schemes.

Today it can be seen that the economic, social and technological areas of business are far less foreseeable and are in a situation that is far less stable than they were a few years ago. This reality has caused, both from a macroeconomic point of view and from a microeconomic point of view, that new paths are being sought for studying situations that on the one hand economic situations and on the other business in general are undergoing. From here it can be seen why Economic Analysis for studying the activity of States and Operative Research for treating business problems has evolved over latter years at a hugely accelerated rate.

With the ever-expanding use of computers there have been spectacular advances in the treatment of data that is available to businesses and with this improve management of the same. Thus this aspect, which is absolutely basic, has to a greater degree been resolved, but a new problem has arisen that derives from obtaining the necessary data. Therefore the problem has been transferred to the search for certain processes that allow for the introduction of determined primary information for its later treatment. Not even resorting to probabilistic models allows for any great advance in certain fields of business activity.

Economists and management scholars have been forced to investigate in this field and have managed to arrive at new schemes that allow for a far more complete consideration of reality, avoiding wherever possible its traditional deformation when resorting to

numerical precision. In these, it is assumed that taking decisions is done within a sphere in which the objectives that are intended to be attained, the limitations to which they are submitted and even the consequences for each one of the alternatives, appear in an imprecise manner.

It has been seen that in order to quantify this imprecision the techniques provided by the theory of probability, and more specifically the theory of decision, are not suitable, because this would imply accepting that the imprecise facts are equivalent to random facts. Within a new framework for decision on business phenomena, imprecision is formalised by assigning to each situation a characteristic function in which a grading is made between total membership and non-membership.

Therefore, while the concept of probability is associated to randomness, the so-called characteristic function of membership is associated to fuzziness. There is a common point between probabilities and the membership function: both one and the other are to be found included between zero and one. Nevertheless, since both concepts have a different origin, they also include different properties. "Uncertain numbers" do not have the same arithmetical rules as "random variables" in the same way as with "confusion in the field of uncertainty"

Works on the problems arising from business management contain a variety of mathematical models the basis for which can be found in certainty or randomness. But as soon as it is verified that the requirements that these hypotheses include give rise to a substantial distancing from the real world, the need was felt to introduce new, more flexible and adaptable schemes. Technological advances, market diversity, multiplicity and product variety, has meant that the intuition of the businessman must be completed by increasingly more complex models. The possibilities that fuzzy sub-sets offer for tackling decision problems in the field of business performance are so wide,

that let there be no doubt, they will be enriching operational techniques for business management.

Traditional schemes, given the impossibility of collating, with any sort of precision, the complex and uncertain reality of businesses, an initial simplification is resorted to in order to be able to carry out later developments based on these simplified elements. The possible initial deviations accumulate and expand as the operative process advances. Also what is lost is certain information from the start that will not be recuperated.

From a new perspective, it is proposed that we include the economical phenomena of businesses with all their imprecision and uncertainty, in order to carry out the pertinent developments maintaining the imprecision (and also all the data) in order to make it "fall" as late as possible. It is always possible (loosing information) to reduce uncertainty.

The possibilities of use in this field are numerous[18] and go from long and short term forecasts, passing through investment selection, stock management, equipment renewal, new product research, multicriteria decisions, personnel selection, creativity, right through to planning with hybrid data, and a very long etcetera. Nevertheless the enthusiasm for these models should not make us forget an unquestionable fact: traditional techniques cannot be consigned to oblivion as they are essential when the phenomena can be measured. But, when the reality of the business creates a whole range of circumstances that cannot be measured, a valuation should be made that is susceptible to being treated by means of fuzzy criteria.

At this time we are going through an era that required realism in the treatment of business management problems, and even though we feel that the time is not ripe for doing without the genius and intuition of the businessman, the complexity of the surroundings in

[18] Kaufmann, A and Gil Aluja, J.: Introducción de la teoría de los subconjuntos borrosos a la gestión de las empresas. Publ. Milladoiro. Santiago de Compostela, 1986.

which we are living causes the requirement for investigating new techniques in order that, through the same, we can continue along the path of progress.

With these techniques an advance will be made towards the formal clarification of business attitudes, increasing in this way the coherence between the evolution of real processes and the schemes drawn up for their treatment. Models will be completed that traditionally have been used for resolving the problems of decision for the businessman, with the appearance of a whole new range of decision techniques that, without a doubt, will allow for fruitful results for the solution to the problems, which are for ever becoming more complex, that the economic activity of business creates.

Bibliography

Barre, R: Prologue to the work by Kaufmann, A and Gil-Aluja, J.: Técnicas operativas de gestión para el tratamiento de la incertidumbre. Publ. Hispano-Europea. Barcelona, 1987.

Gil-Aluja, J.: "La incertidumbre en la economía y gestión de empresas". Proceedings of the IV Congress of the Spanish Association on Technolgy and Fuzzy Logic. Blanes, September 14, 1994.

Gil-Aluja, J.: Lances y desventuras del nuevo paradigma de la teoría de la decisión. Proceedings of the III Congress of the International Society For Management and Fuzzy Economy. Buenos Aires, November 10-13, 1996.

Gil-Aluja J.: "Towards a new paradigm of investment selection in uncertainty". Fuzzy Sets and Systems. Vol. 84, number 2, December 9, 1996.

Gil-Aluja, J.: "Investment in uncertainty". Kluwer Academic Publishers, Dordrecht, 1998.

Gil-Aluja, J.: Elements for a theory of decision in uncertainty". Kluwer Academic Publishers. London, Boston, Dordretch 1999.

Kaufmann, A.: Models mathematiques pour la stimulation inventive. Publ. Albin Michel. Paris, 1979.

Kaufmann, A and Gil Aluja, J.: Introducción de la teoría de los subconjuntos borrosos a la gestión de las empresas. Publ. Milladoiro. Santiago de Compostela, 1986.

Kaufmann, A and Gil Aluja, J.: Modelos para la investigación de efectos olvidados. Ed. Milladoiro. Santiago de Compostela, 1988.

Kaufmann, A and Gil Aluja, J.:Las matemáticas del azar y la incertidumbre. Publ. Ceura, Madrid 1990.

Kaufmann A. and Gil Aluja, J.: Técnicas especiales para la gestión de expertos. Ed. Milladoiro. Santiago de Compostela 1993.

Khun, Thomas: "La estructura de las revoluciones científicas". Ed. Fondo de Cultura Económica, Madrid, 1981.

Popper, Karl: "La lógica de la investigación científica". Ed. Tecnos, Madrid, 1971.

Zahed, L.: "Fuzzy Sets", Information and Control, June 8, 1965.