

Sample Attribute Prototypes of Small and Medium-Sized Manufacturing Firms

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Abstract : This study deals with the identification of a representative sample pattern involved in specific features of SME's. The information was polled from a survey carried out in 1997 among SME's located in Bahía Blanca city (Argentina) and its surroundings. A neural artificial technique clusters a data matrix of p firms characterized by n symmetrical binary variables ($p \gg n$) focusing on some socioeconomic and cultural entrepreneurial aspects. The final decision is reached taking into account the classical first principal direction technique in a complementary way with the rank prompted by the neural artificial classifier. The result suggests an archetype coherent with the characteristics of mainly small seized firms of non-metropolitan areas. Apart from the particular outcomes our contribution struggles for the sake of fostering applications of neural artificial network techniques into the interest of social and economical fields.

Keywords : Discrete neural networks, SME's.

1. INTRODUCTION

We built according to the set theory a neural classifier. This technique allows us to indicate to which extent a particular pattern is compatible with the information of each sample prototype and to select, the archetypal sample standard, in terms of the cardinal number of the equally matching sets associated to the basins of attraction.

The prescreening process proposed is completely automated by a pattern recognizing neural network. In particular, we consider the situation in which the $n \times p$ data array Σ , $\Sigma = [\bar{\mathbf{X}}^1, \bar{\mathbf{X}}^2, \dots, \bar{\mathbf{X}}^p]$ is concerned with Small and Medium Enterprises - SME's - attributes¹. In this case p represents the number of enterprises inquired and n the number of deterministic answers that built up each enterprise rule.

We consider ten answers related with the aspects reflecting the socioeconomic and cultural profile of the entrepreneurs (Table 1, SME's features considered) from a survey carried out in 1997 among SME's located in Bahía Blanca city (Argentina) and its surroundings.

We process the information of those 64 firms between 119 with complete deterministic answers (Yes, No) at the database. The aim addresses the neural technique to sort out the entrepreneurial socioeconomic and cultural archetype inherent to this specific sample. This method performs clusters gathering in a same group all those enterprises that have identical answers in the features considered. Moreover the technique allows us the identification of the pattern with poor sample

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affinity in the analyzed features. Table 2, *Cardinal number of the firms attracted in SME's and Anti-SME's basins*, exhibits the global result rendering by the neural classifier. Table 3, *Cardinal number of the firms attracted in SME's, Anti-SME's basins and in SME's[⊥] crisp set*, shows the results obtained after an operation of undergone re-training for the network by the division of the configuration space in three basins. Finally, Table 4, *Initial and Final Global Cluster Rows*, shows, the neural technique results in detail with the level of matching between each SME pattern and those firms belonging to its basin of attraction in the prominent cases for this sample, i.e., those SME's pattern with higher (*Initial Global Cluster Rows*) and lower nucleation of firms (*Final Global Cluster Rows*) stacked into its respective basins of attraction. The advantage of our approach is also a representational one, enabling us to visualize a three-dimensional sketch of the SME's basins, Figure N° 1 (*Basin of Attraction of SME's Prototype*), as well as sketch the degree of matching and mismatching of each firm inside of the memorized pattern basin by a bi-dimensional draught profile, Figure N° 2 (*Attractivity of SME Prototype*).

The work is organized as follows: First we discuss our inspirational source, Section 2.1 is rooted in a discrete neurodynamics and our formulation for the neural classifier implementation is in Section 2.2; supplemented by novel representational tools we cast our results for a specific SME's data in the Section 3; finally in Section 4, we complement our neural result, with the classical first principal direction technique.

2. NEURAL METHODOLOGY

2.1 The Neural Network Model

A discrete neurodynamics [1] endows a neural artificial classifier by the embodiment of :

- α -level sets into the basins of attraction
- an adequate selection of the learning rule
- a properly operation mode of updating neurons

Our neural classifier considers α as the degree of bit matching between the attractor and those patterns that are trapped inside its basin of attraction.

2.1.1 Discrete State Neural Network

The configuration space is a set of vectors embedded in \mathfrak{R}^n , specifically in the hypercube space of n binary strings $\{-1, +1\}^n$, so $X = \{\vec{\mathbf{x}} \mid \vec{\mathbf{x}} \in \{-1, +1\}^n\}$, herein all $\vec{\xi}$ are provided by polling and storing in a data array that conforms the sample, i.e., an n by p data matrix of binary figures. For mathematical convenience we really treat the deterministic answer Yes $\equiv 1$ and No $\equiv 0$, $b_i = \{1, 0\}$ boolean binary response, as spin binary answer $S_i = \{1, -1\}$, Yes $\equiv 1$ and No $\equiv -1$. The conversion to and from these different binary symmetrical variables is easy via $S_i = 2b_i - 1$ or equivalently by $b_i = \frac{S_i + 1}{2}$.

The matching between two binary spin vectors is reached by means of the Hamming distance. This distance between two binary boolean numbers means the number of bits that are different in the two numbers. In the case of boolean strings $\vec{\mathbf{S}}_b$ and $\vec{\mathbf{x}}_b$ belonging to $\{1, 0\}^n$ space the Hamming distance should be evaluated by :

$$d_H(\vec{\mathbf{s}}_b, \vec{\mathbf{x}}_b) = \sum_{i=1}^n \left[\left[(\vec{\mathbf{s}}_b)_i (1 - (\vec{\mathbf{x}}_b)_i) \right] + \left[(\vec{\mathbf{x}}_b)_i (1 - (\vec{\mathbf{s}}_b)_i) \right] \right]$$

Equivalently for spin strings $\vec{\mathbf{s}}_S$ and $\vec{\mathbf{x}}_S$ belonging to $\{1, -1\}^n$ by :

$$d_H(\vec{\mathbf{s}}_S, \vec{\mathbf{x}}_S) = \frac{1}{4} \sum_{i=1}^n \left\{ \left[((\vec{\mathbf{s}}_S)_i + 1)(1 - (\vec{\mathbf{x}}_S)_i) \right] + \left[((\vec{\mathbf{x}}_S)_i + 1)(1 - (\vec{\mathbf{s}}_S)_i) \right] \right\}$$

Simple algebraic manipulations allow us to reach the relationship between the Hamming distance and the dot product for a couple of spin vectors

$$d_H(\vec{\mathbf{s}}, \vec{\mathbf{x}}) = \frac{n - \langle \vec{\mathbf{x}}, \vec{\mathbf{s}} \rangle}{2}$$

where $\langle \dots \rangle$ represent the ordinary dot product in \mathfrak{R}^n between a pair of spin vectors $\vec{\mathbf{x}}$ and $\vec{\mathbf{s}}$.

This previous equation can be reordered as:

$$\langle \vec{\mathbf{x}}, \vec{\mathbf{s}} \rangle = n - 2 d_H(\vec{\mathbf{x}}, \vec{\mathbf{s}})$$

To pave the way for the definition of alternative synapses of the recurrent neural network of n neurons we arrange every feature chain corresponding to each polled firm into the n by p matrix Σ , $\Sigma = [\vec{\mathbf{x}}^1, \vec{\mathbf{x}}^2, \dots, \vec{\mathbf{x}}^p]$; after conversion of each feature boolean bit into the binary spin one.

Hereafter p will be the sample cardinal number and n the number of deterministic answers that construct the phenotype of each particular firm.

2.1.2 Data processing

The information of the polled enterprises for the identification's sake undergoes a pre-processing datum manipulations. These maneuvers consist in labeling two numbers to every inquired firm belonging to Σ , an ordinal number and a decimal number. The ordinal number (first column at the Global Cluster Rank in Table 2) coincides with the location of the firm in the sample arrange Σ . The decimal number (third column at the Global Cluster Rank in Table 2) corresponds to the unique decimal representation from the n binary boolean number or firm's attributes (second column at the Global Cluster Rank in Table 2). For conversion from binary boolean representation to decimal we merely double the first digit on the left and add the next binary digit, we double again and add the next digit and so on. For instance, $110100010 \equiv 418$, the right hand term in this equivalence is obtained by the following steps: $1 \times 2 + 1 = 3$, $3 \times 2 + 0 = 6$, $6 \times 2 + 1 = 13$, $13 \times 2 + 0 = 26$, $26 \times 2 + 0 = 52$, $52 \times 2 + 0 = 104$, $104 \times 2 + 1 = 209$, $209 \times 2 + 0 = 418$. These two different operational labels for each firm are handled for brevity's sake in the computational tasks.

2.1.3 The Mathematical Model of Single Memory Neural Network

We consider in our application the simple case where there is a single pattern $\vec{\mathbf{s}}^k$ that should be memorized. The condition for this pattern to be stable is just $\text{sgn} \left(\sum_{j=1}^n w_{ij} \mathbf{s}_j \right) = \mathbf{s}_i \quad (\forall i)$ with

$1 \leq i \leq n$, otherwise the update rule $S_i := \text{sgn}\left(\sum_{j=1}^n w_{ij} \mathbf{s}_j\right)$ undergoes changes. It is easy to see that this is true if we take $w_{ij} \propto \mathbf{s}_i \mathbf{s}_j$ since $\mathbf{s}_j^2 = 1$. The constant of proportionality should be $\frac{1}{n}$, for the sake of fostering multiple pattern cases, here n is the number of units (neurons) in this autoassociative network, giving $w_{ij} = \frac{1}{n} \mathbf{s}_i \mathbf{s}_j$ as the linking term between the i -neuron with the j -neuron. Furthermore, if a number (fewer than half) of the bits of the starting state $\vec{\mathbf{x}}$ are wrong (i.e., not equal to $\vec{\mathbf{s}}_i^k$), they will be overwhelmed in the sum for the net input $h_i = \sum_j w_{ij} \mathbf{x}_j$ by the majority that are right, and $\text{sgn}(h_i)$ will still give $\vec{\mathbf{s}}_i^k$. An initial configuration near (in Hamming distance) to $\vec{\mathbf{s}}^k$ will therefore quickly relax to $\vec{\mathbf{s}}^k$. This means that the network will correct errors as desired, and we can say that the pattern $\vec{\mathbf{s}}^k$ is an attractor [2].

Actually, there are two attractors in this simple case, the other one is at $-\vec{\mathbf{s}}^k$ [2]. This is called a reversed state. All starting configurations with more than half the bits different from the original pattern will end up in the reversed state $-\vec{\mathbf{s}}^k$. The configuration space is divided into two basins of attraction.

2.1.4 Autoassociative Memories and Memorization Rule

The synaptic coupling between neurons given by $w_{ij} = \frac{1}{n} \mathbf{s}_i \mathbf{s}_j$ is enhanced if both neurons are active at the same time, this is the Hebbian learning rule that in the simplest form for computational objectives is expressed by $W_{ij} = \frac{1}{n} \sum_{k=1}^{\tilde{p}} \mathbf{s}_i^k \mathbf{s}_j^k$ $\mathbf{s}_i^k = \pm 1$ in the general case of many patterns. Where n is the number of neurons and \tilde{p} is the number of prototype states to be memorized $\{\vec{\mathbf{s}}^k\}$. The complete coupling matrix can be written as $W = \frac{1}{n} \sum_{k=1}^{\tilde{p}} \vec{\mathbf{s}}^k (\vec{\mathbf{s}}^k)^T = \frac{1}{n} \tilde{\Sigma} \tilde{\Sigma}^T$ where $(\vec{\mathbf{s}}^k)^T$ denotes the transpose of $\vec{\mathbf{s}}^k$, $\tilde{\Sigma}$ is the (n, \tilde{p}) matrix whose columns are the vectors $\vec{\mathbf{s}}^k$, $\tilde{\Sigma} = (\vec{\mathbf{s}}^1, \vec{\mathbf{s}}^2, \dots, \vec{\mathbf{s}}^{\tilde{p}})$ and $\tilde{\Sigma}^T$ is the transpose of matrix $\tilde{\Sigma}$.

In the very particular case in which the prototype states are mutually orthogonal, the coupling matrix W , computing for $W = \frac{1}{n} \tilde{\Sigma} \tilde{\Sigma}^T$ is the orthogonal projection matrix (in Euclidean space) into the subspace spanned by the prototype vectors [3].

2.1.5 Analysis of the Network Dynamics

The dynamical behavior of the network which is determined by the components of vectors $W \vec{\mathbf{x}}$; if each component of this vector is zero or has the same sign as the corresponding component of $\vec{\mathbf{x}}$, the state $\vec{\mathbf{x}}$ is stable. In the present particular case (orthogonal prototype states) each vector

$\bar{\mathbf{S}}^k$ is invariant under the projection operation, so that one has $W\bar{\mathbf{S}}^k = \bar{\mathbf{S}}^k$ for all k belong to $\{-1, +1\}^n$ all the prototype states are certain stable states of the system; this property guarantees a perfect retrieval of the stored information.

If the prototype state are not orthogonal, the above property $W\bar{\mathbf{S}}^k = \bar{\mathbf{S}}^k$ for all k is no longer true.

To summarize, Hebbian learning rule is suitable for the design of an associative memory guaranteeing a perfect retrieval of the stored information if the prototype vectors are orthogonal.

A most general prospective show that there exists a coupling matrix which guarantees the stability of a set of prototype vectors, whether correlated or not $W\bar{\mathbf{S}}^k = \bar{\mathbf{S}}^k$ for all k which can be written equivalently as $W\tilde{\Sigma} = \tilde{\Sigma}$ will be the orthogonal projection matrix into the subspace spanned by the prototype vectors family $\{\bar{\mathbf{S}}^k\}$, $W = \tilde{\Sigma}\tilde{\Sigma}^I$ where $\tilde{\Sigma}^I$ is the Moore-Penrose pseudoinverse [4] of $\tilde{\Sigma}$ the relation $W = \tilde{\Sigma}\tilde{\Sigma}^I$ will be termed the *projection rule*. The coupling matrix W , being an orthogonal projection matrix is symmetrical. In the particular case where the prototype $\{\bar{\mathbf{S}}^k\}$

vectors are linearly independent, the synaptic matrix W takes the form $W = \tilde{\Sigma}(\tilde{\Sigma}^T \tilde{\Sigma})^{-1} \tilde{\Sigma}^T$. If the prototypes are arthogonal, the projection rule reduces exactly to the classical Hebb's rule

$$W = \frac{1}{n} \sum_{k=1}^{\tilde{p}} \bar{\mathbf{S}}^k (\bar{\mathbf{S}}^k)^T = \frac{1}{n} \tilde{\Sigma} \tilde{\Sigma}^T \text{ since } (\tilde{\Sigma}^T \tilde{\Sigma})^{-1} = \frac{1}{n} I \text{ where } I \text{ is the identity matrix.}$$

2.1.6 Lyapunov Functions and Storage Capacity

The dynamical properties of the network are provided by using the following Lyapunov function of the state $\bar{\mathbf{x}}$, $E(\bar{\mathbf{x}}) = -\frac{1}{2}(\bar{\mathbf{x}}^T W \bar{\mathbf{x}})$ since W is an orthogonal projection matrix, W is called a projector because $W^2 = W$ and is called an orthogonal projector because in addition $W^T = W$, then the tested pattern energy should be evaluated as $E(\bar{\mathbf{x}}) = -\frac{1}{2} \bar{\mathbf{v}}^T \bar{\mathbf{v}} = -\frac{1}{2} \|\bar{\mathbf{v}}\|^2$, (here

$v_i(t) = \sum_{j=1}^n W_{ij} \bar{\mathbf{x}}_j(t)$); this established that the "energy" is proportional to the square of the synaptic

potential vector $\bar{\mathbf{v}}$. It is possible to show that during the free evolution of the system performing parallel iterations, the energy is an ever decreasing function. Therefore, no cycles can occur [5]. For our practical purposes, we select fully parallel operation that it is more efficient in terms of required computational time. The energy [6] of a prototype state $\bar{\mathbf{S}}^k$ and its $-\bar{\mathbf{S}}^k$ negative is given by $E = -\frac{1}{2} \bar{\mathbf{S}}^{kT} W \bar{\mathbf{S}} = E = -\frac{1}{2} n$. Therefore all prototype states have the same energy, which is the lowest energy possible.

If the prototype states are orthogonal, their attractivity can be evaluated and it is possible to show that any state lying within a Hamming distance of $n/2\tilde{p}$ from a prototype state will

converge to this in one step. Therefore, the minimum number of states attracted by a given

prototype state is given by $\sum_{k=0}^m C_n^k$ where m is the largest integer smaller than $n/2\tilde{p}$.

Thus, the attractivity of the prototype states falls sharply if \tilde{p} becomes of the order $n/2$; correlatively, the number of stable nonprototype states increases.

Since the diagonal coefficients of the projection matrix are smaller than or equal to one, the stability of the prototype states after canceling the diagonal terms is preserved, but their attractivity is altered.

If the prototype states are not orthogonal, no general result can be stated but an order of magnitude of the average minimum attractivity is given by $n/2r$, where r is the rank of the prototype vector family or equivalently $rank(\tilde{\Sigma})$.

2.2 Neural Classifier Implementation

As in all prospective classifications, the number of groups should be known [7] due to the fact that the operational objective is to assign new observations to one of these groups. In pursuit of this the configuration space is divided by $C_{\bar{s}^1}$, and $C_{-\bar{s}^1}$ watersheds, and in the case in which n is an even integer there exist also a crisp set $C_{\bar{s}^1 \perp} = C_{-\bar{s}^1 \perp}$ conformed by all $\vec{x} \in \Sigma$ such that

$\frac{d_H(\vec{x}, \bar{s}^1)}{n} = \frac{d_H(\vec{x}, -\bar{s}^1)}{n} = \frac{1}{2}$, that is those $\vec{x} \in \Sigma$ that are orthogonal to the pattern being memorized, since $\vec{x}^T \vec{z} = n - 2d_H(\vec{x}, \vec{z}) = n - 2 \cdot \frac{n}{2} = 0$.

$C_{\bar{s}^1}$ represents the basin of attraction for the pattern \bar{s}^1 , so $C_{\bar{s}^1} = \left\{ \vec{x} \in \Sigma \mid d_H(\vec{x}, \bar{s}^1) < n/2 \right\}$, here the mismatching degree between the attractor \bar{s}^1 and the tested pattern \vec{x} , that we call $mm_{C_{\bar{s}^1}}(\vec{x})$ with $\vec{x} \in C_{\bar{s}^1}$, is measured taking into account the Hamming distance, by means of the expression $d_H(\vec{x} \in C_{\bar{s}^1}, \bar{s}^1)/n \equiv mm_{C_{\bar{s}^1}}(\vec{x}) \Big|_{\vec{x} \in C_{\bar{s}^1}}$, we made a partition inside of the crisp sets $C_{\bar{s}^1}$ and $C_{-\bar{s}^1}$ for the different values of $d_H(\vec{x} \in C_{\bar{s}^1}, \bar{s}^1)/n$ and we call α -labels or α -levels to the figures that take into account the matching $mm_{C_{\bar{s}^1}}(\vec{x}) \Big|_{\vec{x} \in C_{\bar{s}^1}}$ between the pattern memorize and those patterns that are inside its basin

of attraction by mean of the expression $\alpha = mm_{C_{\bar{s}^1}}(\vec{x}) \Big|_{\vec{x} \in C_{\bar{s}^1}} = 1 - mm_{C_{\bar{s}^1}}(\vec{x}) \Big|_{\vec{x} \in C_{\bar{s}^1}}$ [8, 9] then

inside the respective basins of attraction are built the subsets $C_{\bar{s}^1}^{\alpha} = \left\{ \vec{x} \in C_{\bar{s}^1} \mid \frac{1}{n} d_H(\vec{x}, \bar{s}^1) \leq 1 - \alpha \right\}$ and $C_{-\bar{s}^1}^{\alpha} = \left\{ \vec{x} \in C_{-\bar{s}^1} \mid \frac{1}{n} d_H(\vec{x}, -\bar{s}^1) \leq 1 - \alpha \right\}$, with α belonging to $(0.5, 1]$.

If n is even, the neural-classifier divides the configuration space Σ in three subsets, as follows:

- i) $C_{\bar{s}^1}$ the crisp set of which is the support of all α -subsets $C_{\bar{s}^1}^{\alpha} = \left\{ C_{\bar{s}^1} \mid 0.5 < \alpha \leq 1 \right\}$ or equivalently $C_{\bar{s}^1} = \left\{ \vec{x} \in \Sigma \mid d_H(\vec{x}, \bar{s}^1) < n/2 \right\}$, this basin of attraction is normal by design so $\bigwedge_{\vec{x} \in C_{\bar{s}^1}} d_H(\vec{x} \Big|_{C_{\bar{s}^1}}, \bar{s}^1) = 0$.

- ii) $C_{-\bar{\mathbf{S}}^1}^{\mathbf{a}} = \{C_{-\bar{\mathbf{S}}^1}^{0.5 < \mathbf{a} \leq 1}\} = \{\bar{\mathbf{x}} \in \Sigma \mid d_H(\bar{\mathbf{x}}, -\bar{\mathbf{S}}^1) < n/2\}$ the absence or presence of normality in $C_{-\bar{\mathbf{S}}^1}$ will depend on the information inherent to the data matrix being analyzed.
- iii) $C_{\bar{\mathbf{S}}}^{\perp} = \{\bar{\mathbf{x}} \in \Sigma \mid d_H(\bar{\mathbf{x}}, \bar{\mathbf{S}}^1) = d_H(\bar{\mathbf{x}}, -\bar{\mathbf{S}}^1) = 0.5\}$.

If n is odd the neural-classifier divides the conformation space Σ in two strict subsets without any possibility of $C_{\bar{\mathbf{S}}}^{\perp}$ existence.

The methodology sets the ranking for the global classification upon considering the subsequent cardinal number decreasing into the respective basins of attraction of each embossed pattern (sample firm attribute binary spin vector $\bar{\mathbf{x}}$) and the neural artificial technique selects the sample prototype candidates among those ranked in the first rows beneath the criterion of maximum cardinality for the support of $C_{\bar{\mathbf{x}}}^-$. The basins should be considered as a crisp sets focusing on this first criterion of selection.

Nevertheless, when various patterns fulfill this former pre-screening process a second crible stage should be carried out focusing on any of the previously $C_{\bar{\mathbf{x}}}^-$'s pre-selected basins of attraction considering the cardinal numbers of subsets $C_{\bar{\mathbf{x}}}^{-\alpha}$ therein for the sake of fostering a deeper comparison between the α -label subsets of the different pre-selected candidates . This second sorting process follows the criterion of maximum cardinality in the sequence of subsets $C_{\bar{\xi}_j}^{-\alpha}$ in its respective crisp $C_{\bar{\mathbf{x}}_j}^-$ sets searching comparatively for those with higher cardinal number at the highest α -labels. This computational task is carried out by comparing the number of elements, (i.e., cardinal number #) of the respective subsets $C_{\bar{\mathbf{x}}}^{-\alpha} \subset C_{\bar{\mathbf{x}}}^-$, that is the sequence $\#C_{\bar{\mathbf{x}}_j}^{-1}$, $\#C_{\bar{\mathbf{x}}_j}^{-1-1/n}$, $\#C_{\bar{\mathbf{x}}_j}^{-1-2/n}$, ... , $\#C_{\bar{\mathbf{x}}_j}^{->0.5}$. Therefore, the screening technique will draw out that or those candidates that with the same cardinal number in $C_{\bar{\mathbf{x}}}^-$ will exhibit the higher concentration inside $C_{\bar{\xi}_j}^{-\alpha}$ at the higher α -labels.

Similar criteria enable the methodology to sort out the sample antiprototype sample patterns [10, 11, 12, 13], that are the poorer associated sample pattern, it is worth to notice that there is no guarantee of uniqueness in both screening proposed process. These candidates should be selected from the final rows of the global classification (Table 2 and Table 3) following similar preceding trials, but now inside the α -level sets of the respective antipattern basins of attraction of the candidates $C_{-\bar{\xi}_j}^-$, by searching for major gathering of firms inside of the anti-pattern sample items, in the concerning with our application the search is realized for major stacked firms inside of $C_{-\bar{\xi}_j}^-$ Anti-SME's patterns basins. In spite of those cases where $C_{-\bar{\xi}_j}^-$'s basins have lack of normality in the sample been considered. The presence or absence of normality in anti-pattern vectors of attributes is a characteristic that depend intrinsically on the sample data to deal with.

Meanwhile, the technique also identifies the redundant information of all those enterprises being similarly characterized for the specially considered attributes as well as this neural screener will

explicit the normality absence or presence in antipattern sample attractors. Both revelations are of very practical usefulness.

3. APPLICATION to SME's ATTRIBUTES

The results of the application to SME's deterministic attributes selected from the database of Small and Medium Manufacturing Enterprises survey over Regional allocated Firms are gathered in Table 2 (*Cardinal number of the firms attracted in SME's and Anti-SME's basins*), Table 3 (*Cardinal number of the firms attracted in SME's, Anti-SME's basins and SME's[⊥] crisp sets*) and Table 4 (*Initial and Final Complete Global Cluster Rows*). From the one hundred and nineteen firms surveyed, we selected only those which had complete answers in the symmetric binary responses selected. Table 1, was concerned with the sociocultural aspects of the owners that were selected for the sake of revealing the sample profile of the entrepreneurs' attitudes, motivations and behavior with regard to their interests in its marketplace and their perspectives to further management strategies as decision takers.

Table 1
SME's features considered

<i>Attribute</i>	<i>Information revealed</i>
v_1	<i>Does the firm belong to an enterprise holding ?</i>
v_2	<i>Owners. Are they relatives ?</i>
v_3	<i>Are partners college graduates?</i>
v_4	<i>Executive management, is it a task shared by the owners ?</i>
v_5	<i>Does the firm perform special tasks required ?</i>
v_6	<i>Did the firm import foreign articles for local trade ?</i>
v_7	<i>Did the enterprise invest from 1991 onwards ?</i>
v_8	<i>Concerning the 1985/1989 period, was the investment level of 1991/1996 period higher or lower ?</i>
v_9	<i>Know how. Does the firm receive systematical or sporadical updated information ?</i>
v_{10}	<i>Is the firm technically assisted by Development and Research National Institutions ?</i>

For the sake of fostering the applications, a neural artificial network should be constructed with as many neurons as attributes and the unsupervised learning rule in which the single fundamental memory is each one of the items. In this particular situation the items are the enterprises polled and characterized by a binary pattern of ten deterministic Yes $\equiv 1$ or No $\equiv -1$ answers. The neural technique was applied really over sixty four firms, those with complete answers to the SME's survey. Table 2, shows the results of a global neuro-classification. The first column shows the ordinal position of the enterprise in the datum matrix of $n \times p$ dimension ($n = 10$ attributes; $p = 64$ enterprises).

The second column entitled by "Firm's attributes" contains the binary vector of ten components corresponding to the deterministic answers of the respective firm. Third column encloses the decimal number that is said to be a 1-1 (one-to-one) mapping with second column. Meanwhile the forth column contains the decimal number biunivocally associated with the Anti-SME pattern, i.e., the decimal number corresponding to the pattern belonging to the second column but with opposite sign in each component. In the fifth column the figure indicates the cardinality of the basin of attraction from each respective row SME firm. Column sixth shows the amount of firms attracted inside of the Anti-SME basin. As in all prospective classifications the number of groups

should be known due to the fact that the operational objective is to assign new observations to one of these groups. Here the neural classifier decides between the SME pattern memorized and the Anti-SME pattern in term of level \mathbf{a} of bit matching with each new firm presented to the network for the cases in which $\mathbf{a} \geq 0.5$ the enterprise belongs to the SME basin otherwise to the Anti-SME basin.

Table 2
Cardinal number of the firms attracted in SME's and Anti-SME's basins

<i>Ordinal Number of the Firm location in Σ</i>	<i>Firm's attributes</i>	<i>Decimal representation of SME</i>	<i>Decimal representation of Anti-SME</i>	<i>Cardinal Number inside of SME's basin</i>	<i>Cardinal Number inside of Anti-SME's basin</i>
40	- 1 1 - 1 1 1 - 1 1 - 1 1 - 1	362	661	64	0
3	- 1 1 - 1 1 1 - 1 1 1 1 - 1	366	641	63	1
41	- 1 1 - 1 1 1 - 1 1 1 1 - 1	366	641	63	1
50	- 1 1 - 1 1 1 - 1 1 1 1 - 1	366	641	63	1
4	- 1 1 1 - 1 1 - 1 1 - 1 1 - 1	426	597	62	2
17	- 1 1 - 1 1 1 - 1 1 1 - 1 - 1	364	659	62	2
56	- 1 1 - 1 1 1 - 1 1 1 - 1 - 1	364	659	62	2
33	- 1 1 - 1 - 1 1 - 1 1 1 1 - 1	238	721	62	2
62	- 1 1 - 1 - 1 1 - 1 1 1 1 - 1	238	721	62	2
2	1 1 - 1 1 1 - 1 1 - 1 1 1	363	660	61	3
52	- 1 - 1 - 1 1 1 1 - 1 1 - 1 1 - 1	106	917	61	3
31	- 1 1 - 1 1 - 1 - 1 1 - 1 1 - 1	330	693	60	4
10	- 1 1 1 - 1 1 - 1 1 - 1 1 1 - 1	430	593	59	5
12	- 1 - 1 - 1 - 1 1 - 1 1 1 1 - 1	46	977	59	5
21	- 1 - 1 - 1 - 1 1 - 1 1 1 1 - 1	46	977	59	5
18	- 1 1 - 1 1 - 1 - 1 1 1 - 1 - 1	332	691	59	5
20	- 1 1 - 1 1 - 1 - 1 1 1 - 1 - 1	332	691	59	5
28	- 1 1 - 1 1 - 1 - 1 1 1 - 1 - 1	332	691	59	5
24	- 1 1 - 1 - 1 1 - 1 1 1 - 1 - 1	300	723	59	5
27	- 1 1 - 1 - 1 1 - 1 1 1 - 1 - 1	300	723	59	5
63	- 1 1 1 1 1 - 1 1 1 1 1 1	495	528	59	5
13	- 1 1 - 1 1 - 1 - 1 1 - 1 - 1 - 1	640	695	58	6
35	- 1 1 - 1 1 - 1 - 1 1 - 1 - 1 - 1	640	695	58	6
26	- 1 1 - 1 1 - 1 - 1 - 1 - 1 1 - 1	322	701	58	6
49	- 1 1 - 1 1 1 - 1 - 1 - 1 - 1 - 1	704	671	58	6
51	- 1 - 1 - 1 1 1 - 1 1 1 1 1 1	111	912	58	6
57	- 1 1 - 1 - 1 - 1 - 1 1 1 1 - 1	270	753	58	6
7	- 1 1 1 1 1 - 1 - 1 1 1 1 - 1	462	561	57	7
14	- 1 - 1 - 1 1 - 1 - 1 1 1 1 - 1	78	945	57	7
23	- 1 1 - 1 - 1 - 1 - 1 1 1 - 1 - 1	268	755	57	7
34	- 1 1 - 1 1 1 1 1 - 1 1 1 1	379	644	57	7
37	- 1 1 - 1 - 1 1 1 1 - 1 1 - 1	314	709	57	7
39	- 1 1 1 - 1 1 - 1 1 1 - 1 - 1	428	595	57	7
42	- 1 1 - 1 - 1 1 - 1 - 1 - 1 1 - 1	290	733	57	7
54	- 1 1 - 1 - 1 1 - 1 - 1 - 1 1 - 1	290	733	57	7
25	- 1 - 1 - 1 1 - 1 - 1 1 - 1 1 - 1	74	949	56	8
61	- 1 1 - 1 - 1 1 - 1 1 - 1 - 1 - 1	297	726	56	8
8	- 1 1 1 1 - 1 - 1 1 1 1 1 1	463	560	55	9
58	- 1 1 1 1 1 - 1 - 1 1 1 1 1	487	536	55	9
11	- 1 1 1 1 - 1 - 1 1 1 - 1 - 1	460	563	54	10
1	- 1 1 1 1 - 1 - 1 1 1 - 1 - 1	460	563	54	10
15	- 1 1 - 1 1 - 1 - 1 1 1 - 1 1	333	690	54	10
16	- 1 - 1 - 1 1 1 - 1 1 - 1 - 1 1	105	918	54	10
53	- 1 - 1 - 1 1 1 - 1 1 - 1 1 1	107	916	54	10
59	- 1 1 1 - 1 1 - 1 - 1 1 1 - 1	422	601	54	10
5	1 1 1 1 - 1 - 1 - 1 - 1 1 - 1	450	413	53	11
9	- 1 - 1 1 - 1 - 1 - 1 1 1 1 - 1	102	881	53	11
38	- 1 1 - 1 - 1 1 - 1 - 1 - 1 1 1	289	734	53	11
46	- 1 1 1 - 1 1 1 1 1 1 - 1 - 1	442	513	52	12
55	- 1 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	128	767	52	12
64	- 1 1 - 1 - 1 - 1 - 1 - 1 - 1 1 1	259	764	52	12
19	- 1 - 1 - 1 1 1 1 1 - 1 - 1 - 1	120	583	51	13
22	- 1 - 1 - 1 - 1 1 - 1 - 1 - 1 1 - 1	34	989	51	13
32	- 1 1 - 1 1 - 1 1 1 1 - 1 - 1	348	675	51	13
36	- 1 1 - 1 - 1 1 - 1 - 1 - 1 1 1	259	732	51	13
43	- 1 - 1 1 1 1 1 - 1 - 1 - 1 - 1	98	797	51	13
48	- 1 - 1 1 - 1 - 1 - 1 - 1 - 1 - 1	320	703	51	13
30	- 1 - 1 1 1 1 - 1 1 1 - 1 1	237	786	50	14

47	-	1	-	1	1	1	-	1	1	1	1	239	784	50	14
6	-	1	1	1	1	1	-	1	-	1	-	496	527	48	16
60	-	1	1	1	-	1	1	1	1	-	1	444	579	46	18
44	-	1	1	1	-	1	1	1	1	1	1	447	576	44	20
29	1	1	1	1	-	1	-	1	1	1	1	974	49	35	29
45	-	1	-	1	1	1	1	-	1	-	1	243	780	32	32

The advantage of our approach is also a representational one, because enabling us to visualize in three-dimensional coordinates all conformations of the sample attracted inside each specific memorized firm. Figure 1, *SME Prototype Basin of Attraction*, shows in one coordinate edge the decimal rank of the firm's attributes, at the second coordinate edge are located the neighborhood of the firm being memorized and at the third coordinates edge are labeled the different levels of matching. Meanwhile the process of sifting out is performed by the neural classifier on the screen of the computer displaying a spot if the new enterprise presented to the net is attracted inside the basins of the firm being memorized. Simultaneously, Figure 2, a two-dimensional sketch highlights which is the enterprise trapped (*Upper frame, Attractivity of the SME Prototype*) and the degree of matching with the firms being considered as the memorized pattern (*Lower frame, a-level of the firms trapped*).

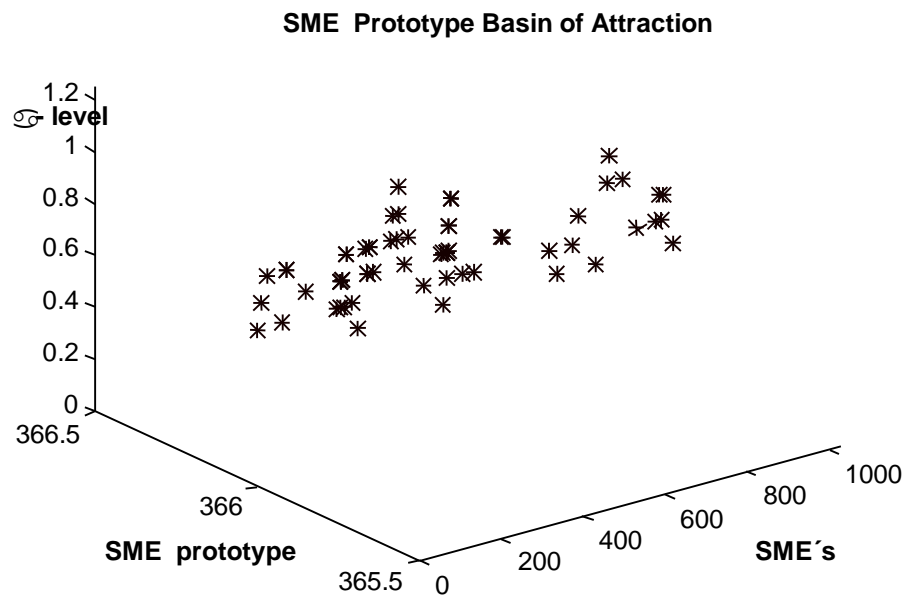


Figure N °1. Basin of Attraction of SME's Prototype

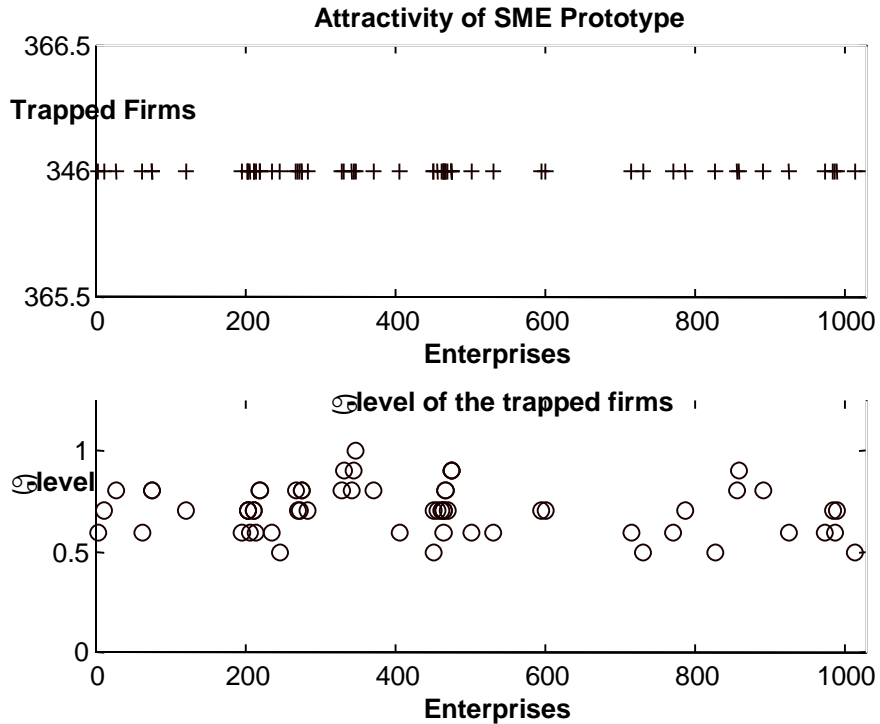


Figure N °2. Profiles of Attractivity and α -levels of SME Archetype

The neural methodology sets the global rank in Table 2 upon considering the sequential increase in the cardinal number of the Anti-SME basins.

At a first glance the firms that are more related to the other in the sample are located at the initial rows of the Table 2 (e.g., 40, 3, 41, 50, 4, 17), on the contrary those with poorer nucleation are placed at the final rows (60, 44, 29, 45). An inspection into the third column enables us the identification of all firms with the same attribute profile (e.g., 3=41=50, 17=56, 33=62, and so forth) and consequently the same location at the global cluster. Furthermore, for this special sample the mismatching between the decimal number inside the third and the fourth columns assures the lack of any Anti-SME pattern inside this sample, i. e., the absence of normality in each Anti-SME basin.

The previous analysis requires a re-learning neural classifier process in order to acquire a major sample knowledge. This new neural task was performed by sifting out the enterprises into three distinguishable groups, those that belong to the SME basin with bit matching from $\mathbf{a} > 0.5$ to $\mathbf{a} \leq 1$, those that are attracted by the Anti-SME basin with $0 < \mathbf{a} < 0.5$ and those that have the same amount the bit in matching even though mismatching with the SME pattern, that is with $\mathbf{a} = 0.5$, this firms belong to the orthogonal crisp set of SME basin, SME^\perp .

The network outcomes are showed in Table 3, *Cardinal number of the firms in SME's, Anti-SME's basins and SME^\perp crisp sets*. Here the last column embraces the amount of enterprises that not belong to the SME basin neither to the Anti-SME basin, in other words, these firms have the half of answers in coincidence with whatever the SME pattern or the Anti-SME pattern, for this reason these enterprises are gathered in a crisp basin apart.

Table 3
Cardinal number of the firms attracted in SME's, Anti-SME's basins
and SME^\perp crisp sets

Ordinal items	Firm's attributes										SME	Anti-SME	SME's basin	Anti-SME's basin	SME [⊥] basin
40	-1	1	-1	1	1	-1	1	-1	1	-1	362	661	59	0	5
3=41=50	-1	1	-1	1	1	-1	1	1	1	-1	366	641	59	1	4
52	-1	-1	-1	1	1	-1	1	-1	1	-1	106	917	49	1	14
4	-1	1	1	-1	1	-1	1	-1	1	-1	426	597	51	2	11
17=56	-1	1	-1	1	1	-1	1	1	-1	-1	364	659	55	2	7
33=62	-1	1	-1	-1	1	-1	1	1	1	-1	238	721	55	2	7
2	-1	1	-1	1	1	-1	1	-1	1	1	363	660	53	2	9
31	-1	1	-1	1	-1	-1	1	-1	1	-1	330	693	53	4	7
10	-1	1	1	-1	1	-1	1	1	1	-1	430	593	49	4	11
12=21	-1	-1	-1	-1	1	-1	1	1	1	-1	46	977	46	4	14
18=20=28	-1	1	-1	1	-1	-1	1	1	-1	-1	332	691	46	5	13
24=27	-1	1	-1	-1	1	-1	1	1	-1	-1	300	723	46	5	13
63	-1	1	1	1	1	-1	1	1	1	1	495	528	46	5	13
13=35	-1	1	-1	1	-1	-1	1	-1	-1	-1	640	695	48	5	11
51	-1	-1	-1	1	1	-1	1	1	1	1	111	912	42	5	17
26	-1	1	-1	1	-1	-1	-1	-1	1	-1	322	701	46	6	12
49	-1	1	-1	1	1	-1	-1	-1	-1	-1	704	671	46	6	12
57	-1	1	-1	-1	-1	-1	1	1	1	-1	270	753	52	6	6
39	-1	1	1	-1	1	-1	1	1	-1	-1	428	595	46	6	12
7	-1	1	1	1	-1	-1	1	1	1	-1	462	561	48	7	9
14	-1	-1	-1	1	-1	-1	1	1	1	-1	78	945	45	7	12
23	-1	1	-1	-1	-1	-1	1	1	-1	-1	268	755	45	7	12
34	-1	1	-1	1	1	1	1	-1	1	1	379	644	39	7	18
37	-1	1	-1	-1	1	1	1	-1	1	-1	314	709	44	7	13
42=54	-1	1	-1	-1	1	-1	-1	-1	1	-1	290	733	43	7	14
25	-1	-1	-1	1	-1	-1	1	-1	1	-1	74	949	48	8	8
61	-1	1	-1	-1	1	-1	1	-1	-1	1	297	726	46	8	10
8	-1	1	1	1	-1	-1	1	1	1	1	463	560	42	9	13
58	-1	1	1	1	1	-1	-1	1	1	1	487	536	38	9	17
1=11	-1	1	1	1	-1	-1	1	1	-1	-1	460	563	44	9	11
15	-1	1	-1	1	-1	-1	1	1	-1	1	333	690	44	9	11
16	-1	-1	-1	1	1	-1	1	-1	-1	1	105	918	38	10	16
53	-1	-1	-1	1	1	-1	1	-1	1	1	107	916	42	10	12
59	-1	1	1	-1	1	-1	-1	1	1	-1	422	601	42	10	12
5	-1	1	1	1	-1	-1	-1	-1	1	-1	450	413	39	11	14
9	-1	-1	1	-1	-1	-1	1	1	1	-1	102	881	38	11	15
48	-1	1	-1	1	-1	-1	-1	-1	-1	-1	320	703	42	11	11
38	-1	1	-1	-1	1	-1	-1	-1	-1	1	289	734	31	12	21
46	-1	1	1	-1	1	1	1	1	1	-1	442	513	35	12	17
64	-1	1	-1	-1	-1	-1	-1	-1	1	1	259	764	30	12	22
55	-1	1	-1	-1	-1	-1	-1	-1	-1	-1	128	767	35	13	16
19	-1	-1	-1	1	1	1	1	-1	-1	-1	120	583	37	13	14
22	-1	-1	-1	-1	1	-1	-1	-1	1	-1	34	989	40	13	11
36	-1	1	-1	-1	1	-1	-1	-1	1	1	259	732	39	13	12
43	-1	-1	1	1	1	-1	-1	-1	1	-1	98	797	35	13	16
30	-1	-1	1	1	1	-1	1	1	-1	1	237	786	35	13	16
32	-1	1	-1	1	-1	1	1	1	-1	-1	348	675	37	14	13
47	-1	-1	1	1	1	-1	1	1	1	1	239	784	35	15	14
60	-1	1	1	-1	1	1	1	1	-1	-1	444	579	34	16	14
6	-1	1	1	1	1	1	-1	-1	-1	-1	496	527	25	18	21
44	-1	1	1	-1	1	1	1	1	1	1	447	576	31	19	14
29	1	1	1	1	-1	-1	1	1	1	-1	974	49	32	19	13
45	-1	-1	1	1	1	1	-1	-1	1	1	243	780	19	32	13

Table3 shows the new neural classifier results, *Cardinal number of firms attracted in SME's, Anti-SME's basins and SME[⊥] crisp sets*, these figures clear up a step forward in our searching for a sample standard or phenotype of higher nucleation and as well as poorer related firm pattern. The candidates to sample prototype or sample phenotype are the firms with ordinal number 40 or 3 (= 41 = 50) and with attribute profile 362 and 366 respectively. On the other hand, in this particular sample is not necessary go further for recognizing that the poorer nucleation candidate is the firm with ordinal number 45 and its attribute profile is 243. Even though, previously to take our final decision in both objectives our technique has the possibility of scour more deeper considering the cardinal number of the sets at the different level inside each basin of the candidates prompted by the results in the Table 3.

Table 4, Initial and Final Complete Global Cluster Rows, shows in detail the initial and final complete global clustering rows as a summarized draft of the whole landscape classification obtained by the neuro-technique.

Table 4
Initial and Final Complete Global Cluster Rows

	Initial Global Cluster Rows	
α -level in SME's basin	α -level in Anti-SME's basin	α -level = 0.5 in SME's basin
Ordinal number {° Decimal} Basin's cardinal numbers and members	Ordinal number {° Decimal} Basin's cardinal numbers and members	Orthogonal basin's cardinal numbers and members
<p>40 {° 362} # 59</p> <p>I: { 40 }; # 1 0.9:{ 2, 3 ° 41 ° 50, 31, 52 }; # 6 0.8:{ 4, 13 ° 35, 56 ° 17, 49, 25, 42 ° 54, 26, 37, 33 ° 62, 34, 53 }; # 15 0.7:{ 5, 7, 10, 21 ° 12, 14, 16, 61, 48, 43, 36, 27 ° 24, 18 ° 20 ° 28, 19, 22, 57, 51, 63 }; # 21 0.6:{ 58, 8, 15, 29, 32, 6, 64, 38, 1 ° 11, 39, 55, 46, 47, 59, 23 }; # 16</p>	<p>661 # 0</p> <p>{ }</p>	<p>#5</p> <p>0.5: {9, 30, 44, 45, 60}</p>
<p>3= 41= 50 {° 366} # 59</p> <p>I: { 3 ° 41 ° 50 }; # 3 0.9:{ 17 ° 56, 33 ° 62, 40 }; # 5 0.8:{ 31, 24 ° 27, 10, 14, 12 ° 21, 7, 20 ° 18 ° 28, 63, 2, 51, 52, 57 }; # 16 0.7:{59, 42 ° 54, 46, 47, 39, 13 ° 35, 8, 49, 26, 23, 1 ° 11, 58, 25, 15, 4, 34, 29, 37, 32, 53} # 23 0.6:{61,22,36,48,9,30,5,16,19,60,44,43} # 12</p>	<p>641 # 1</p> <p>{ 45 }</p>	<p>#4</p> <p>0.5: {6,38,55,64}</p>
<p>52 {° 106} # 49</p> <p>I: { 52 }; # 1 0.9:{40,25,53} # 3 0.8:{51,50,22,21,12,19,3,16,2,1,14,43,31,41} # 14 0.7:{62,56,49,35,34,13,37,33,47,4,17,42,26,54} # 14 0.6:{61,45,48,63,36,30,18,9,10,20,5,28,27,5,57} # 16</p>	<p>917 # 1</p> <p>{44}</p>	<p>#15</p> <p>0.5:{59,38,55,32,6,11,58,15,8,23,64,23,29,46,39}</p>
<p>4 {° 426} # 50</p> <p>I: { 4 }; # 1 0.9:{10} # 1 0.8:{37,39,46,54,40,59,33,62,42} # 9 0.7:{52,43,63,27,60,7,9,41,5,12,50,3,2,36,61,31,21,57,22,24,44} # 21 0.6:{53,38,55,35,25,64,23,58,17,8,13,6,47,56,26,34,29,49} # 18</p>	<p>597 # 2</p> <p>{15,32 }</p>	<p>#12</p> <p>0.5: {30,48,20,19,18, != 11, 28,45,51,16,14}</p>

	Final Global Cluster Rows	
<p style="text-align: center;">60 { ° 444 } # 34</p> <p>1: { 60 }; # 1 0.9: { 39, 46 }; # 2 0.8: { 10, 27 = 24, 44 }; # 4 0.7: { 4, 17 = 56, 62 = 33, 23, 37, 1 = 11, 59, 32, 6 }; # 12 0.6: { 12 = 21, 3 = 41 = 50, 18 = 20 = 28, 63, 57, 7, 61, 9, 19, 30 }; # 15</p>	<p style="text-align: center;">579 # 16</p> <p>1: { }; # 0 0.9: { }; # 0 0.8: { 64 }; # 1 0.7: { 52, 26, 25, 53 }; # 4 0.6: { 2, 31, 51, 14, 16, 5, 48, 22, 36, 43, 45 }; # 11</p>	<p style="text-align: center;">#14</p> <p>0.5: { 40, 13 = 35, 49, 34, 42 = 54, 8, 58, 15, 38, 55, 47, 29 }; # 14</p>
<p style="text-align: center;">29 { ° 974 } # 36</p> <p>1: { 29 }; # 1 0.9: { 7 }; # 1 0.8: { 8, 1 = 11 }; # 3 0.7: { 57, 41 = 50 = 3, 10, 9, 31, 28 = 18 = 20, 5, 63, 14 }; # 13 0.6: { 59, 58, 13 = 35, 17 = 56, 62 = 33, 25, 15, 26, 40, 4, 32, 47, 39, 46, 23 }; # 18</p>	<p style="text-align: center;">49 # 16</p> <p>1: { }; # 0 0.9: { }; # 0 0.8: { 38 }; # 1 0.7: { 16, 19, 22, 36, 45, 61 }; # 6 0.6: { 6, 34, 37, 54 = 42, 49, 53, 55, 64 }; # 9</p>	<p style="text-align: center;">#12</p> <p>0.5: { 60, 52, 43, 48, 51, 30, 24 = 27, 2, 12 = 21, 44 }; # 12</p>
<p style="text-align: center;">44 { ° 447 } # 31</p> <p>1: { 44 }; # 1 0.9: { 46 }; # 1 0.8: { 10, 60, 63 }; # 3 0.7: { 4, 59, 62 = 33, 58, 47, 39, 37, 8, 34 }; # 10 0.6: { 7, 30, 3 = 41 = 50, 51, 9, 57, 24 = 27, 2, 45, 61, 36, 12 = 21 }; # 16</p>	<p style="text-align: center;">576 # 16</p> <p>1: { }; # 0 0.9: { }; # 0 0.8: { 48 }; # 1 0.7: { 55, 49, 13 = 35 }; # 4 0.6: { 52, 43, 31, 18 = 20 = 28, 22, 19, 16, 14, 5 }; # 11</p>	<p style="text-align: center;">#13</p> <p>0.5: { 17 = 56, 53, 29, 1 = 11, 6, 23, 42 = 54, 64, 15, 38 } # 13</p>
<p style="text-align: center;">45 { ° 243 } # 18</p> <p>1: { 45 }; # 1 0.9: { }; # 0 0.8: { 43 }; # 1 0.7: { 47, 6, 34, 58, 53 }; # 5 0.6: { 19, 16, 52, 36, 5, 44, 63, 30, 2, 22, 51 }; # 11</p>	<p style="text-align: center;">780 # 33</p> <p>1: { }; # 0 0.9: { 23, 57 }; # 2 0.8: { 28 = 20 = 18, 27 = 24 }; # 5 0.7: { 62 = 33, 56 = 17, 55, 39, 35 = 13, 32, 29, 15, 1 = 11 }; # 13 0.6: { 61, 60, 50 = 3 = 41, 48, 31, 21 = 12, 14, 10, 9, 7 }; # 13</p>	<p style="text-align: center;">#13</p> <p>0.5: { 4, 37, 42, 26, 46, 54, 59, 64, 40, 49, 38, 8, 25 } # 13</p>

For the sake of comparison the cardinal numbers of the subsets inside its respective basins we draw off the two first rows of the *Initial Global Cluster* of the Table 4. These figures allows us to built up Table 5., Cardinal Numbers of the the α -subsets inside the Basins of Attraction of both prominents sample prototype profile. Both candidates have the same storage capacity (59), but the upper level of matching are more crowded for the pattern attribute fulfill for the three firms $3 = 41 = 50$, as another evidence the subsets of $\mathbf{a}=0.6$, are less concentrated while the superior levels $\mathbf{a}=0.7$, $\mathbf{a}=0.8$ have more concentration than in the pattern labeled by 40. In addition, the normality of basin is fulfill by three members of the sample. Then the third screening process taking into account the higher level inside the subsets of the candidates extracted from Table 3. allows in this sample select as sample phenotype the pattern fulfill by the firms $3 = 41 = 50$.

Table 5
Cardinal Numbers of the \mathbf{a} -subsets inside the Basins of Attraction of both prominents sample prototype profile

$\# C_{\bar{s}=40} = 59$ $\# C_{-\bar{s}=40} = 0$ $\# C_{\bar{s}^\perp} = \# C_{-\bar{s}^\perp} = 5$	$\# C_{\bar{s}=3=41=50} = 59$ $\# C_{-\bar{s}} = 1$ $\# C_{\bar{s}^\perp} = \# C_{-\bar{s}^\perp} = 4$
$\# C_{\bar{s}=40} = 59$	$\# C_{\bar{s}=3=41=50} = 59$
$\# C_{\bar{s}=40}^{a=1} = 1$	$\# C_{\bar{s}=3=41=50}^{a=1} = 3$
$\# C_{\bar{s}=40}^{a=0.9} = 6$	$\# C_{\bar{s}=3=41=50}^{a=0.9} = 5$
$\# C_{\bar{s}=40}^{a=0.8} = 15$	$\# C_{\bar{s}=3=41=50}^{a=0.8} = 16$
$\# C_{\bar{s}=40}^{a=0.7} = 21$	$\# C_{\bar{s}=3=41=50}^{a=0.7} = 23$
$\# C_{\bar{s}=40}^{a=0.6} = 16$	$\# C_{\bar{s}=3=41=50}^{a=0.6} = 12$

Similar analysis, considering the *Final Global Cluster* of the Table 4 allows us the selection of the candidates with poorer nucleation. In this particular sample, the candidate are practically draw off from Table 3, after the second neural sorting process. Table 6, *Cardinal Numbers of the a-subsets inside the Basins of Attraction of poorer nucleation sample candidate profiles*, shows the firm labeled as 45 is the poorer nucleation pattern of the SME sample.

Table 6
Cardinal Numbers of the a-subsets inside the Basins of Attraction
of poorer nucleation sample candidate profiles

$\# C_{\bar{s}=45} = 18$	$\# C_{-\bar{s}=-45} = 33$	$\# C_{\bar{s}}^{a=0.5} = 13$
$\# C_{\bar{s}=45} = 18 : \{ 45, 43, 47, 6, 34, 58, 53, 19, 16, 52, 36, 5, 44, 63, 30, 2, 22, 51 \}; \# 18$	$\# C_{-\bar{s}=-45} = 33 : \{ 23, 57, 18 = 20 = 28, 24 = 27, 33 = 62, 17 = 56, 55, 39, 35 = 13, 32, 29, 15, 1 = 11, 61, 60, 50 = 3 = 41, 48, 31, 21 = 12, 14, 10, 9, 7 \}; \# 33$	$\# C_{\bar{s}}^{a=0.5} = 13 : \{ 4, 37, 42, 26, 46, 54, 59, 64, 40, 49, 38, 8, 25 \}; \# 13$
$\# C_{\bar{s}=45}^{a=1} = 1$ $= \{ 45 \}$	$\# C_{-\bar{s}=-45}^{a=1} = 0$ $: \{ 0 \}; \# 0$	
$\# C_{\bar{s}=45}^{a=0.9} = 0$ $= \{ \}$	$\# C_{-\bar{s}=-45}^{a=0.9} = 2$ $: \{ 23, 57 \}; \# 2$	
$\# C_{\bar{s}=45}^{a=0.8} = 1$ $= \{ 43 \}$	$\# C_{-\bar{s}=-45}^{a=0.8} = 5$ $: \{ 18 = 20 = 28, 24 = 27 \}; \# 5$	
$\# C_{\bar{s}=45}^{a=0.7} = 5$ $= \{ 34, 58, 53, 47, 6 \}$	$\# C_{-\bar{s}=-45}^{a=0.7} = 13$ $: \{ 17 = 56, 33 = 62, 13 = 35, 39, 1 = 11, 15, 55, 32, 29 \}; \# 13$	

$\# C_{\bar{s}=45}^{a=0.6} = 8$ = { 52, 63, 51, 16, 5, 19, 36, 44 }	$\# C_{-\bar{s}=-45}^{a=0.6} = 13$: { 61, 60, 50 = 3 = 41, 48, 31, 12 = 21, 14, 10, 9, 7 }; # 13	
--	--	--

$\# C_{\bar{s}=29} = 36$	$\# C_{-\bar{s}=-29} = 16$	$\# C_{\bar{s}}^{a=0.5} = 12$
$\# C_{\bar{s}=29} = 36$	$\# C_{-\bar{s}=-29} = 16$	$\# C_{\bar{s}}^{a=0.5} = 12$: { 60, 52, 43, 48, 51, 30, 27 = 24, 2, 12 = 21, 44 }; # 12
$\# C_{\bar{s}=29}^{a=1} = 1$: { 29 }; # 1	$\# C_{-\bar{s}=-29}^{a=1} = 0$: { }; # 0	
$\# C_{\bar{s}=29}^{a=0.9} = 1$: { 7 }; # 1	$\# C_{-\bar{s}=-29}^{a=0.9} = 0$: { }; # 0	
$\# C_{\bar{s}=29}^{a=0.8} = 3$: { 8, 1 = 11 }; # 3	$\# C_{-\bar{s}=-29}^{a=0.8} = 1$: { 38 }; # 1	
$\# C_{\bar{s}=29}^{a=0.7} = 13$: { 57, 50 = 3 = 41, 10, 9, 31, 18 = 20 = 28, 5, 63, 14 }; # 13	$\# C_{-\bar{s}=-29}^{a=0.7} = 6$: { 16, 19, 22, 36, 45, 61 }; # 6	
$\# C_{\bar{s}=29}^{a=0.6} = 18$: { 59, 58, 13 = 35, 17 = 56, 25, 62 = 33, 15, 26, 40, 4, 32, 47, 39, 46, 23 }; # 18	$\# C_{-\bar{s}=-29}^{a=0.6} = 9$: { 6, 34, 37, 54 = 42, 49, 53, 55, 64 }; # 9	

$\# C_{\bar{s}=44} = 31$	$\# C_{-\bar{s}=-44} = 18$	$\# C_{\bar{s}}^{a=0.5} = 15$
$\# C_{\bar{s}=44} = 31$: { 44, 46, 10, 60, 63, 4, 59, 33 = 62, 58, 47, 39, 37, 8, 34, 7, 30, 3 = 50 = 41, 51, 9, 57, 2, 45, 61, 27 = 24, 36, 12 = 21 }; # 31	$\# C_{-\bar{s}=-44} = 18$	$\# C_{\bar{s}}^{a=0.5} = 15$: { 17 = 56, 53, 29, 6, 1 = 11, 23, 42 = 54, 64, 15, 38, 32, 40 }; # 15
$\# C_{\bar{s}=44}^{a=1} = 1$: { 44 }; # 1	$\# C_{-\bar{s}=-44}^{a=1} = 0$: { 0 }; # 0	
$\# C_{\bar{s}=44}^{a=0.9} = 1$: { 46 }; # 1	$\# C_{-\bar{s}=-44}^{a=0.9} = 0$: { 0 }; # 0	
$\# C_{\bar{s}=44}^{a=0.8} = 3$: { 10, 60, 63 }; # 3	$\# C_{-\bar{s}=-44}^{a=0.8} = 1$: { 48 }; # 1	
$\# C_{\bar{s}=44}^{a=0.7} = 10$: { 4, 59, 62 = 33, 58, 47, 39, 37, 8, 34 }; # 10	$\# C_{-\bar{s}=-44}^{a=0.7} = 6$: { 55, 49, 13 = 35, 25, 26 }; # 6	
$\# C_{\bar{s}=44}^{a=0.6} = 16$: { 7, 30, 41	$\# C_{-\bar{s}=-44}^{a=0.6} = 11$: { 52,	

= 50 = 3, 51, 9, 57, 2, 45, 61, 24 = 27, 36, 12 = 21 }; # 16	43, 31, 18 = 20 = 28, 22, 19, 16, 14, 5 }; # 11	
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Subsection 3.1 (*Attributes of the SME's archetype*) and 3.2 (*Attributes of SME's poorest associated*) contain the patterns selected by the neural classifier, after translation of the decimal representation into the original economic variables involving the deterministic answer.

3.1 Attributes of the SME's archetype

The prototype arising from SME's manufacturing firms selected under the consideration of the features shown in Table 1 is the following binary pattern with ten components: $\vec{\sigma} = [-1 \ 1 \ -1 \ 1 \ 1 \ -1 \ 1 \ 1 \ 1 \ -1]$, ($\# C_{\vec{\sigma}=3=41=50} = 59$, $\# C_{\vec{\sigma}=-3} = 1$, $\# C_{\vec{\sigma}}^{a=0.5} = 4$, Table 4 and 5) which in terms of the information examined indicates that these firms abide by: v_1 : Do not belong to an entrepreneurial group; v_2 : The owners are relatives; v_3 : The entrepreneurs formal education is not above High School degree; v_4 : The owners share management issues; v_5 : The enterprises do special tasks required; v_6 : The companies do not import foreign products; v_7 : The firms made investments from 1991 onwards; v_8 : The investments during the 1991/96 period were higher to those made during 85/89 period; v_9 : The enterprises do systematical reception of technological innovative ideas and v_{10} : The enterprises do not acquire technological transference at scientific and/or technological domestic development centers.

This outcome corresponds to the labeled firms in the second row of global classification Table 3, detected by the designed technique as those enterprises that have a high attractive basin and high concentration at the superior α -level crisp basin set. Particularly, the archetype is fitted into three firms with 3, 41 and 50 as sample ordinal numbers.

This enterprises correspond to: 3: A manufacturer of aluminium's openings. 41: Tailor's shop. 50: Hamburger's manufacturer.

3.2 Attributes of SME's poorest associated prototype

The less representative bahiense SMEs pattern, i. e., that with least sample nucleation ($\# C_{\vec{\sigma}=45} = 18$, $\# C_{\vec{\sigma}=-45} = 33$, $\# C_{\vec{\sigma}}^{a=0.5} = 13$, Table 4 and Table 6) according to the methodology proposed here is taken from the final row of the global cluster (Table 2, Table 3) and its sample ordinal label is 45 and its binary code univocally is associated to the decimal figure 828 equivalent to the following string of deterministic attributes $\{-1 \ -1 \ 1 \ 1 \ 1 \ 1 \ -1 \ -1 \ 1 \ 1\}$. Then, this prospective profile is wrapped up in: v_1 : Do not belong to an entrepreneurial group; v_2 : The owners are not relatives; v_3 : The entrepreneurs formal education is university level; v_4 : The owners share management issues; v_5 : The enterprises do special tasks required; v_6 : The companies import foreign products for local market trading; v_7 : The firms made investments from 1991 onward; v_8 : The investments in the 1991/96 period were inferior to those in the 85/89 period; v_9 : The enterprises do systematical reception of technological innovative ideas and v_{10} : The enterprises acquire technological transference at the scientific and technological development domestic centers.

This enterprise correspond to the ordinal number 45: Air-conditioned installation.

4. OVERLAPPING BETWEEN CLASSICAL AND NEURAL CONTEXTS

Then the final decision is obtained taking into account the sample's first principal direction under the guise to reinforce the forecasted results obtained by the neural classifier. This objective

will be overtaken once the area overlapped within both classical and neural contexts had been recognized. The geometry of the first principal direction allows us the use of both contexts as complementary techniques [14, 15]. The reliability of this proposal lie upon the following paragraph considerations.

We can sensibly define the *first principal direction* as the direction \bar{a} ($\bar{a}'\bar{a}=1$) which minimizes the angles \mathbf{q}_i between \bar{a} and \bar{I}_i , $i=1, 2, 3, \dots, p$ be the sample of unitary vectors; that is, if $Q = \sum_{i=1}^p (\cos \mathbf{q}_i)^2 = \sum_{i=1}^p (\bar{a}'\bar{I}_i)^2$ is maximum. The square of the angle ensures that we are talking of angles between two lines \bar{a} and \bar{I}_i passing through the origin, so that for this purpose each \bar{I}_i is considered as an axis rather than a direction. Let $T = \sum_{i=1}^p \bar{I}_i \bar{I}_i'$ be the matrix of sums of squares and products (Notice that the mean has not subtracted off). Suppose that I_1, I_2, \dots, I_n are the eigenvalues of T , ($I_1 > \dots > I_n$) and $\bar{\mathbf{g}}_1, \bar{\mathbf{g}}_2, \dots, \bar{\mathbf{g}}_n$ are the corresponding eigenvectors. In view of $\bar{I}_i \bar{I}_i' = 1$, $tr(T) = n = I_1 + \dots + I_n$. However, note that I_n is specified from $tr T = \sum_{i=1}^n I_i =$

n as soon as. Further maximizing $Q = \sum_{i=1}^p (\cos \mathbf{q}_i)^2 = \sum_{i=1}^p (\bar{a}'\bar{I}_i)^2$ is equivalent to maximizing $Q = \bar{a}'T\bar{a}$ subject to $\bar{a}'\bar{a}=1$. This optimization problem come from these considerations $\sum_{i=1}^p \bar{a}'\bar{I}_i \bar{a}'\bar{I}_i = \sum_{i=1}^p \bar{a}'\bar{I}_i \bar{I}_i' \bar{a} = \bar{a}' \sum_{i=1}^p \bar{I}_i \bar{I}_i' \bar{a} = \bar{a}' T \bar{a}$, here $T = \sum_{i=1}^p \bar{I}_i \bar{I}_i'$ and \bar{a} should be satisfied $\bar{a}'\bar{a}=1$. Using the argument of principal component analysis, we conclude that the maximizing value of \bar{a} is $\bar{a} = \bar{\mathbf{g}}_1$, which represents the *first principal direction*, and $\max Q = I_1$, which measures the *concentration* around this axis. If $I_1 = I_2 = \dots = I_n$, the angular distances will be almost the same for all \bar{a} . Hence, in this case, the sample distribution is approximately uniform. This optimization problem has unique solution when T has full rank and all eigenvalues are distinct real positive numbers. Particularly our sample fulfill this requirement.

Table 7. shows the matrix T , meanwhile the Table 8. and Table 9. shows the eigenvectors and the eigenvalues of this matrix.

Table 7, Matriz T

5.0391	-1.9597	1.2398	-0.8398	-1.4397	3.0394	-1.8397	0.2399	-1.0398	1.4397
-2.2396	4.7592	-0.4399	-0.3600	-0.1600	-1.4397	0.6399	0.1600	-0.1600	-1.4397
1.2398	-0.1600	5.0391	-0.2399	0.3600	1.2398	-0.4399	2.0396	0.7599	0.4399
-0.8398	-0.6399	-0.2399	5.0391	-0.7599	-0.8398	0.8398	-0.4399	0.0400	0.3600
-1.4397	-0.4399	0.3600	-0.7599	5.0391	0.1600	0.2399	-0.6399	0.6399	0.9599
3.0394	-1.1598	1.2398	-0.8398	0.1600	5.0391	-1.0398	-0.1600	-1.0398	1.0398
-1.8397	0.3600	-0.4399	0.8398	0.2399	-1.0398	5.0391	2.1596	0.2399	-0.6399
0.1200	0.3199	1.9197	-0.3199	-0.5199	-0.2800	2.2796	4.9191	-0.1200	-0.2800
-1.0398	-0.4399	0.7599	0.0400	0.6399	-1.0398	0.2399	-0.2399	5.0391	0.1600
1.4397	-1.1598	0.4399	0.3600	0.9599	1.0398	-0.6399	-0.1600	0.1600	5.0391

**Table 8
Eigenvectors of the Matriz T**

0.5681	-0.0567	0.7524	0.0935	-0.1838	0.0953	0.0114	-0.1742	0.0810	0.0505
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-0.3807	-0.0255	0.2404	-0.0490	-0.1989	-0.3948	0.4103	0.5440	0.0157	0.2757
0.2097	-0.5165	-0.0200	0.4613	0.2085	-0.1435	-0.1558	0.4290	0.3326	-0.1595
-0.1221	0.0495	0.1306	-0.2338	-0.0204	0.7649	0.0945	0.3960	0.1000	-0.3022
-0.0331	0.0905	0.3341	-0.1752	0.6585	-0.2845	-0.1401	0.0858	-0.4465	-0.2906
0.4948	-0.0491	-0.3722	-0.2616	-0.0267	-0.0750	0.6448	-0.0659	-0.2118	-0.4117
-0.3420	-0.4325	0.2323	0.4681	-0.0026	0.2488	0.3811	-0.3207	-0.4028	-0.1026
-0.0849	-0.7235	-0.1526	-0.5995	-0.0990	0.0195	-0.2340	-0.0402	-0.0978	0.1977
-0.1216	-0.0486	0.0584	-0.1854	0.5231	-0.0072	0.4012	-0.3347	0.6263	0.0153
0.2976	0.0586	-0.1668	0.0966	0.4062	0.2839	0.0441	0.3236	-0.2535	0.7077

Table 9
Eigenvalues of the Matriz T

11.7704	0	0	0	0	0	0	0	0	0
0	7.7555	0	0	0	0	0	0	0	0
0	0	1.0747	0	0	0	0	0	0	0
0	0	0	1.2789	0	0	0	0	0	0
0	0	0	0	6.9003	0	0	0	0	0
0	0	0	0	0	6.0700	0	0	0	0
0	0	0	0	0	0	2.7651	0	0	0
0	0	0	0	0	0	0	3.8695	0	0
0	0	0	0	0	0	0	0	5.0174	0
0	0	0	0	0	0	0	0	0	3.4894

The eigenvector associated with the biggest eigenvalue is the first principal direction, its projection in the hypercube space of spin binary vectors $\{-1, 1\}^{10}$ is coincident with our forecasted vector as the sample prototype by the neural methodology.

-0.5681	-1
0.3807	1
-0.2097	-1
0.1221	1
0.0331	1
-0.4948	-1
0.3420	1
0.0849	1
0.1216	1
-0.2976	-1

CONCLUSIONS

In this work, we present a neural classifier to deal with a sample of symmetrical binary variables, based on the concept of similarity. This neural-methodology characterized the sample archetype and the poorer associated pattern of a sample of SME's attributes. The specific application of the neural classifier to a sample of deterministic answer polled on Small and Medium-Sized Manufacturing Enterprises located in Bahía Blanca city (Argentina) and its surroundings sifts out a socioeconomic feature profile compatible with the prototype of non-metropolitan areas.

The computational resources of the neural artificial technique enable us to rank each enterprise in the entrepreneurial assembly in term of matching or mismatching between binary spin digits, the detection of those firms that are sharing the same feature string gets rid with all the redundant information, our proposal distinguishes if there is absence or presence of normality in the anti-enterprise pattern and recognizes those firms with even matching as mismatching with the firm being memorized. These previous issues pave the way to forecast our final decision about if it is possible draws off a sample phenotype.

For the sake of comparison between a classical technique and our neural artificial method, we compute the projected first principal direction into the $\{1, -1\}^{10}$ hypercube space of a particular matrix built up with the sample data, beneath the support of the geometric idea inherent of the first principal direction concept.

Our contribution is a humble footprint fostering the interrelationship between social areas and the neuroartificial fields of what concern to education and research goals.

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