Analyzing Inhomogeneous Planar and rectangular Waveguides by Galerkin's Method.

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Abstract

The simulation of the response of device used in Integrated optics at a given excitation needs numerical methods. These methods must be precise and economical from memory size and time points of view .

According to these criteria, we have developed a software based of Galerkin's Method for determining the modal characteristics of Inhomogeneous planar and rectangular waveguides.
In this paper, the calculations results for effective indices and field distributions obtained with our software (Galerkin's method) are presented and compared with other methods.

1-Introduction

The conception and optimization of the components utilized in Integrated optics require the development defined for isotropic medium by [1] :

$$\nabla^2\vec{E} + k_0^2\varepsilon_r\vec{E} + \vec{\nabla}\left(\vec{E}\vec{\nabla}\varepsilon_r\right) = \vec{0} \qquad (1)$$

$$\nabla^2\vec{H} + k_0^2\vec{H} + \frac{\vec{\nabla}\varepsilon_r}{\varepsilon_r} \wedge \left(\vec{\nabla} \wedge \vec{H}\right) = \vec{0}$$

Where:

∇^2 is the scalar laplacian if the fields is expressed in Cartesian systems.
$k_0 = \omega\sqrt{\varepsilon_0\mu_0} = 2\pi/\lambda_0$: is the free space wave number.
$\varepsilon_r = n^2$ denote the refractive index profile.

Over the years, several numerical methods have been developed to solve the wave equations. Among the most popular techniques are :

1-The finite element and finite difference methods[2] ; the cross section is divided into simple polygons, where the field is expressed in terms of polynomial functions over these polygons, and zero elsewhere. These methods can be applied to a wide range of optical waveguide structures but the principal inconvenient is the appearance of spurious modes.

2-Approached methods; these methods are based on the separation of the variables (Marcatili and effective indices methods) [3,4], these techniques consists in the conversion for a two dimensional to a one dimensional problem. However, these approaches are applied only for scalar resolution and some simple geometry's.

3-The method that we have proposed is the Galerkin's method, which is based on expanding the electromagnetic field in Fourier series. It has been applied to circular fibers than to slab waveguides. Generally, compared with other methods, the development Fourie series is easy, rapid, and doesn't generate spurious mode.

2-Mathematical formulation

The Galerkin's method is based on expanding the electromagnetic field in terms of an orthonormal set of functions to convert victorial or scalar wave equations on matrixes equations on eigenvalues[5,6].

We consider a 3-D optical waveguide, where x and y are the transverse directions with refractive index profile n(x,y), and z is the propagation direction. The vector wave equation (eq 1) is given as [1].

$$\nabla^2 E + k_0^2 n^2(x,y)E + \nabla\left(E\frac{\nabla n^2(x,y)}{n^2(x,y)}\right) = 0 \qquad (2)$$

A similar vector wave equation is obtained for magnetic field [5], for this reason only the electric vector wave equation is considered in this paper and converted to a system of coupled equation :

$$\nabla_t^2 E_x + (k_0^2 n^2 - \beta^2)E_x + 2\frac{\partial}{\partial x}\left(E_x \frac{\partial \ln n}{\partial x} + E_y \frac{\partial \ln n}{\partial y}\right) = 0 \qquad (3)$$

$$\nabla_t^2 E_y + (k_0^2 n^2 - \beta^2)E_y + 2\frac{\partial}{\partial y}\left(E_y \frac{\partial \ln n}{\partial y} + E_x \frac{\partial \ln n}{\partial x}\right) = 0$$

According the GALERKIN's method , the transverse electric components are expanded on a suitable series of known functions $\{\phi_\mu, \phi_\eta\}$.

$$E_x = \sum_{\mu=1}^{Nx}\sum_{\eta=1}^{Ny} A_{\mu\eta}\phi_{\mu\eta}(x,y)$$

$$(4)$$

$$E_y = \sum_{\mu=1}^{Nx}\sum_{\eta=1}^{Ny} B_{\mu\eta}\bar{\phi}_{\mu\eta}(x,y)$$

Where Nx and Ny are the number of functions necessary for development .
The coefficient $A_{\mu\ \eta}$,$B_{\mu\ \eta}$ are initially unknown.

The basis functions orthonormality condition is defined by :

$\iint_D \phi_{\mu\eta}(x,y)\phi_{\mu'\eta'}(x,y)dxdy = \delta_{\mu\eta\mu'\eta'}$ .Where D is the definition domain of basis functions .

A simple and convenient complete orthonormal set consists of sine functions
If we expand the unknown field as sine functions, it is necessary to enclose the transverse dimensions of wave guide in a sufficiently large domain to ensure that the electromagnetic field is negligible at the boundary. We take :

$$\phi_{\mu\eta}(x,y) = \frac{2}{\sqrt{LxLy}}.\sin\frac{\mu\pi}{Lx}x\sin\frac{\eta\pi}{Ly}y \quad \text{avec } \mu = 1,\ldots\ldots\ldots \text{et } \eta = 1,\ldots\ldots\ldots$$

Within $0\leq x \leq Lx$ and $0\leq y \leq Ly$

-By replaced the fields Ex and Ey in system (eq3) by their series expansions (eq4).
-Multiplication with basis functions and integration ; we obtained the system of coupled equations :

$$\sum_{\mu=1}^{Nx}\sum_{\eta=1}^{Ny}(M_{\mu'\eta'\mu\eta}A_{\mu\eta} + N_{\mu'\eta'\mu\eta}B_{\mu\eta}) = \left(\frac{\beta}{k_0}\right)^2 A_{\mu'\eta'}$$

$$(5)$$

$$\sum_{\mu=1}^{Nx}\sum_{\eta=1}^{Ny}(R_{\mu'\eta'\mu\eta}A_{\mu\eta} + S_{\mu'\eta'\mu\eta}B_{\mu\eta}) = \left(\frac{\beta}{k_0}\right)^2 B_{\mu'\eta'}$$

With the matrix elements :

$$M_{\mu'\eta'\mu\eta} = \frac{4}{L_x L_y}\int_0^{Lx}dx\int_0^{Ly}dy\left[\begin{array}{l}\left(n^2(x,y) - \frac{\sigma_\mu^2 + \rho_\eta^2}{k_0^2}\right)S_\mu(x)S_{\mu'}(x)S_\eta(y)S_{\eta'}(y) + \\ 2\frac{\sigma_{\mu'}}{k_0^2}\ln(n)\left(\begin{array}{l}\sigma_\mu C_\mu(x)C_{\mu'}(x)S_\eta(y)S_{\eta'}(y) - \\ \sigma_{\mu'}S_\mu(x)S_{\mu'}(x)S_\eta(y)S_{\eta'}(y)\end{array}\right)\end{array}\right]$$

$$N_{\mu'\eta'\mu\eta} = \frac{8\sigma_{\mu'}}{k_0^2 L_x L_y}\int_0^{Lx}dx\int_0^{Ly}dy\left[\ln(n)\left(\begin{array}{l}\overline{\rho_\eta}\bar{S}_\mu(x)C_{\mu'}(x)\bar{C}_\eta(y)S_{\eta'}(y) + \\ \rho_{\eta'}\bar{S}_\mu(x)C_{\mu'}(x)\bar{S}_\eta(y)S_{\eta'}(y)\end{array}\right)\right]$$

$$R_{\mu'\eta'\mu\eta} = \frac{8\overline{\rho_{\eta'}}}{k_0^2 L_x L_y}\int_0^{Lx}dx\int_0^{Ly}dy\left[\ln(n)\left(\begin{array}{l}\sigma_\mu C_\mu(x)\bar{S}_{\mu'}(x)S_\eta(y)\bar{C}_{\eta'}(y) + \\ \overline{\sigma_{\mu'}}S_\mu(x)\bar{C}_{\mu'}(x)S_\eta(y)\bar{C}_{\eta'}(y)\end{array}\right)\right]$$

$$S_{\mu'\eta'\mu\eta} = \frac{4}{L_x L_y}\int_0^{Lx} dx \int_0^{Ly} dy \left[\left(n^2(x,y) - \frac{\bar{\sigma}_\mu^2 + \bar{\rho}_\eta^2}{k_0^2} \right) \bar{S}_\mu(x)\bar{S}_{\mu'}(x)\bar{S}_\eta(y)\bar{S}_{\eta'}(y) + 2\frac{\overline{\rho_{\eta'}}}{k_0^2}\ln(n) \left(\begin{array}{l} \overline{\rho_\eta}\bar{S}_\mu(x)\bar{S}_{\mu'}(x)\bar{C}_\eta(y)\bar{C}_{\eta'}(y) - \\ \overline{\rho_{\eta'}}\bar{S}_\mu(x)\bar{S}_{\mu'}(x)\bar{S}_\eta(y)\bar{S}_{\eta'}(y) \end{array} \right) \right]$$

$$\text{where}: \sigma_\mu = \frac{\pi\mu}{Lx}, \qquad s_\mu(x) = \sin(\sigma_\mu x), \qquad c_\mu(x) = \cos(\sigma_\mu x)$$

M,S are a matrixes corresponding to the scalar wave equation, and N and R represent the coupling terms
These equations can be written in conventional matrix form by defining a vector X consisting of elements $A_{\mu\eta}$,$B_{\mu\eta}$
And by also defining a matrix C composed of the coefficients $M_{\mu\eta}$, $N_{\mu\eta}$, $R_{\mu\eta}$, $S_{\mu\eta}$.
The equation systems can now be written in form of eigenvector X and eigenvalue $(n_{eff}=\beta/k_0)^2$;
(n_{eff} : is the effectif index) $\qquad C.X = n_{eff}^{\ 2}\, X$;

3-Results et discussions :

planar waveguide

In the first part we are applied ours software for study the guiding modes in inhomogeneous planar wave guide :

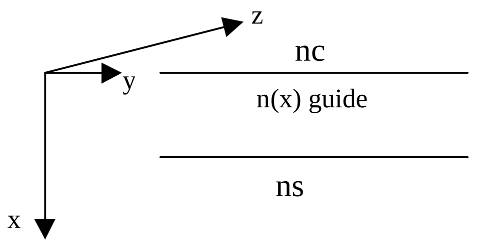


Figure (1) waveguide with refractive index n(x)

we considered asymmetric graded-index guides with refractive index profile n(x) of the form :

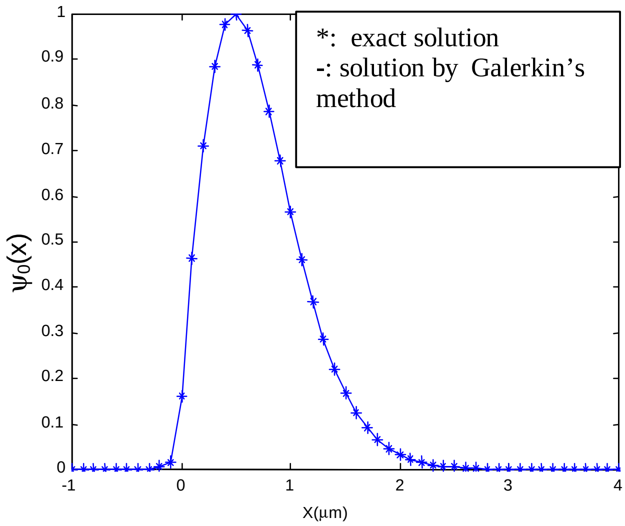
$$n^2(x) = [\, n_s + \Delta f(x)\,]^2 \qquad x \geq 0$$
$$\quad = n_c^2 \qquad x < 0$$

Where :
n_c :refractive index of superstrate =1.00 (Air)
n_s : refractive index of substrate =2.177.
Δ: measure of increase in the refractive index, which provides waveguiding action =0.09837
f(x) : the profile shape function, to compared ours results, we presented here the exponential profile:

f(x)= $e^{-\frac{x}{a}}$ (a : breaking factor of the function).
a=2.22726µm ,λ_0 =0.6328µm.

	Exact	WKB	Galerkin
0	2.24267	2.24289	2.24264
1	2.22153	2.22159	2.22150
2	2.20735	2.20738	2.20732
3	2.19714	2.19715	2.19651
4	2.18967	2.18968	2.18519

Table(1)



Figure(2): Plot of modal field for m=0 versus distance at the enclosed domain
L x=5µm, -1µm <x<4µm

This table (table 1) exposed the propagated modes to planar guide with comparison of finite element (exact) [6,7], WKB[3,7], and Galerkin mode spectra neff calculated for Nx=50 and Lx=5µm.

Generally speaking, Galerkin's method gives more accurate results than the WKB. On the other hand, the effective index of higher-order are in considerable error. That explain one-self of fact the artificial width Lx=5µm is not sufficient to let passed the higher-order modes, for reamed we are widened the domain of n(x) variation.

The field of lowest order mode $\psi_0(x)$ in the case Lx=5µm (figure(2) is indistinguishable from exact solution [7]

Micro wave guide

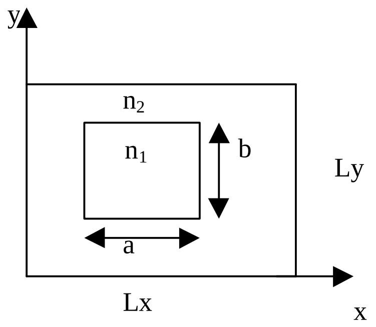


Figure (3) :the structure of rectangular Waveguide

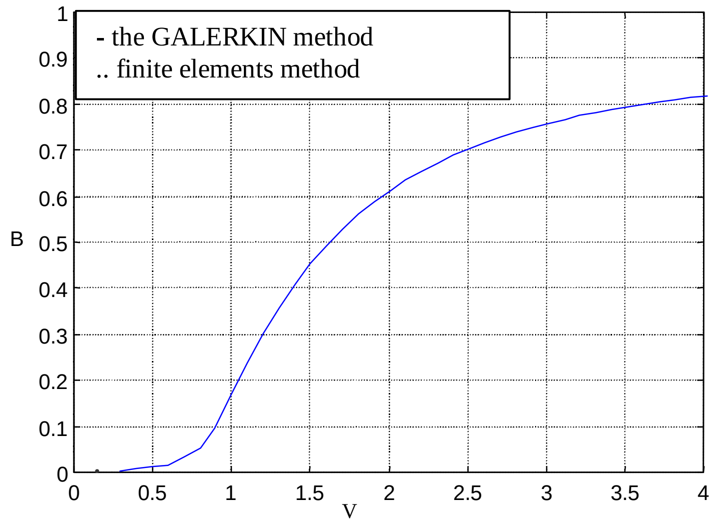


Figure 4: normalized propagation constant B of fundamental mode with normalized frequency for square waveguide (a/b=1), (n_1-n_2=0.5)

we define the normalized propagation constant B with : $B = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2}$

The normalized frequency V: $V = \frac{k_0}{\pi} b\left(n_1^2 - n_2^2\right)^{1/2}$

We are calculated the normalized propagation constant of fundamentally mode for square wave guide with normalized frequency from victories solutions using 20x20 basis functions figure 4 .Their are a good accord with results obtained with finite elements method [Goell 69].

The accuracy and validity of the standard scalar approximation are further examined for the fundamental mode in a square step-index waveguide, as shown in figure 5 :

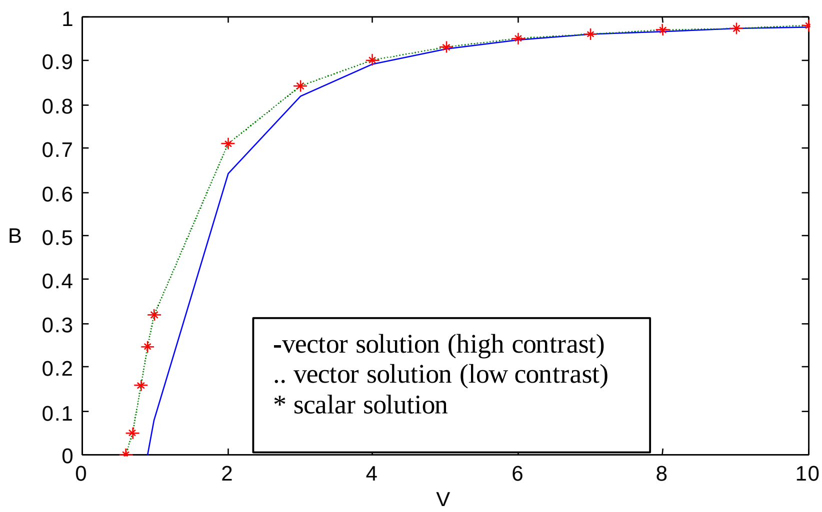


Figure5: the Values of B(V)for fundamentally mode of square waveguide V_x=V_y =V

High contrast: n_1=2 n_2 =1 ; Low contrast: n_1=1.01 n_2 =1

For low contrast in refractive index the scalar approximation leads to accurate results over the entire V range. In case of high contrasts, however , the scalar and vector solutions are significantly different for small V numbers. As expected, the scalar approximation approaches the vector solution with increasing V number due to more complete mode confinement.

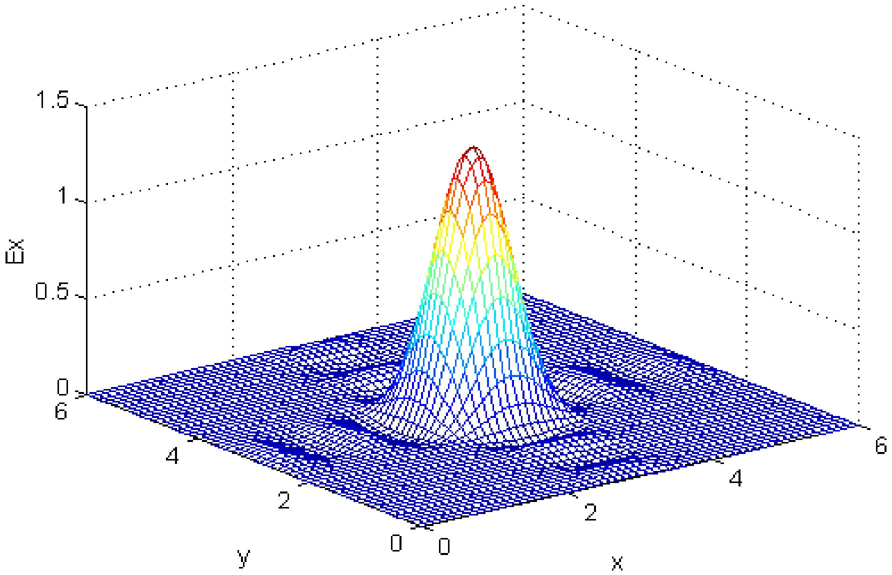


Figure 6 : Repartition of the field Ex (x, y)(Nfilm=1.605; Nsubstrat=1.515; a=b=3㎛ m, l =0.6328㎛ m)

The repartition of electromagnetic field as shown in figure6 illustrate the Confinement of guided mode in the film.

4-Conclusion

In this present paper we have made the Galerkin's method applied for inhomogeneous waveguides. The prediction of the modal field distribution through the trigonometric function expansion supplied accuracy results with a good choice of enclosed region Lx and Ly.
In all, cases however no spurious mode solutions in the range ns<neff<ns+Δ have been observed.

Therefore this method present the essential advantages without the others methods and in particular to the finite element or effective index method.

1-Generallity and aptitude of the method to treat a differences structures (victories, inhomogeneous) [1,8].
2-Simplicity in analyzing.
3-No spurious mode.

In the end, this method can be directly used for analyzing the propagation from no uniform optics system's according the propagation direction.
In particular, association the Galerkin method to characteristic matrix permit to treat the periodic structures presented a fort discontinuity of refraction index from the direction of propagation.
Since, we are able to say ,that we disposed the preferment implement for the conception of Itegrated optic components

5-References

[1] A.Weisshaar, J.Li, R.L.Gallawa, and I.C.Goyal *IEEE GHT wave technology*, Vol 13(1996), No8, P1795-1800.
[2] C.Vassallo,"1993-1995 Optical Mode Solvers", O*pt.Quant.Electron.*, Vol 29(1997),No 2,P.95-114.
[3] E.A.J.Marcatili and A.A.Hardy , *IEEE J.Quant.Electron.*, Vol 24(1988),P.766-774.
[4] K.S.Chiang, , *IEEE Trans On Microwave Theory and Tech*, Vol 44(1996),No 5, P. 692-700.
[5] D.Marcuse , , IEEE J.Quant.Electron., Vol 28(1992),No 2,P.459-465.
[6] S.Y.Wang, W.Y.Lee and S.J.Hwang , *IEEE Photonics Technol.Lett.*,Vol 5(1993),n°12,P.1439-1441.
[7] Janta.J, and J.Ctyroky , *Opt.comm*, vol 25(1978), P49-52
[8] J.P.Spyridon and U.K.Nikolaos , *IEEE Trans on Microwave Theory and Tech*, Vol 44(1996),No 5, P. 641-650.